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An Iterative Control for Optimally Managing Export Limits of Distributed Generators

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SUMMARY
Managing active power output of Distributed Generators (DG), also called Flexible Interconnection Capacity Solutions (FICS), has the potential to significantly increase the utilization of distribution system assets and allow for more nameplate DG to be interconnected without causing overloading. This type of DG management must be able to 1) effectively consider one or multiple distribution system constraints simultaneously, 2) accommodate a variety of overall objective functions (e.g., order of curtailment, minimize overall resource curtailment, cost minimization, etc.), and 3) account for pre-existing binding contractual agreements.
This paper introduces a novel quadratic programming approach that iteratively solves for the optimal dispatch to overcome the nonlinear and discrete nature of a distribution system. This type of analysis can be used to estimate the operational impact that would be experienced by a DG under a FICS scheme and inform project modeling and investment decisions for a developer to adopt a FICS scheme. Additionally, the approach can be extended to other types of Distributed Energy Resources (DER), such as energy storage and demand response, for scheduling applications that involves time-dependent constraints.

KEYWORDS

Flexible interconnection capacity solution, active network management, distributed generator, optimal curtailment, energy storage

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I. INTRODUCTION

Conventional hosting capacity analysis (HCA) of a distribution system is typically based upon expected operating scenarios to achieve a safe and reliable system operation while assuming no direct utility control of DGs. While this allows the interconnected distributed generators to operate in an unconstrained manner, it may also result in under-utilized system assets and potentially higher interconnection or distribution system upgrade costs.

Flexible interconnection capacity solution (FICS)[1] takes into consideration the fact that hosting capacity is time-variant and dependent on system operating conditions. Through real time curtailment of DGs, it has the potential to accommodate significantly increased nameplate rating of distributed generation without violating system operation constraints. Most distribution system constraints are only encountered during a small number of hours in a year and, therefore, the energy curtailed is small compared to the overall annual energy production. This type of targeted curtailment to mitigate system constraints is not uncommon for transmission connected renewable resources. This approach enables greater overall energy production from DGs during their operating lifetime.

A FICS control can be as simple as an export limit applied to a single DG unit based on either its local or a remotely monitored measurement to address system constraints (e.g., the thermal rating of an upstream line section). However, when multiple DG units are connected at different locations across the feeder system and contributing differently to a variety of operational constraints, developing a curtailment strategy that fairly and effectively controls active power may be challenging. One aspect that is often included in designing a proper curtailment control logic is the contractual agreement between a utility and the DG owner. At the stage of flexible interconnection application and approval, a priority order may be assigned to the DGs to establish a rule of curtailment that specifies how each DG shall be curtailed when needed by the utility operator. Fig. 1 provides a simple illustration of two such rules that are commonly referred are Last-In, First-Out (LIFO) and Pro-Rata [3]. More commercial arrangement options are briefly discussed in [4].

In a LIFO scheme, the DGs are to be curtailed in a pre-defined priority order based on when a DG project has reached a specific milestone in the interconnection process. A DG will not be curtailed until all DGs with lower priority are fully curtailed. In the example, DG-2 will not be curtailed until DG-3 is fully curtailed, and DG-1 will not be curtailed until DG-2 is fully curtailed.

In contrast, all DGs in a Pro-Rata scheme share the same priority and are curtailed by the same percentage of a reference base, which can be either the nameplate rating or the present output power of individual DG based on established agreement.

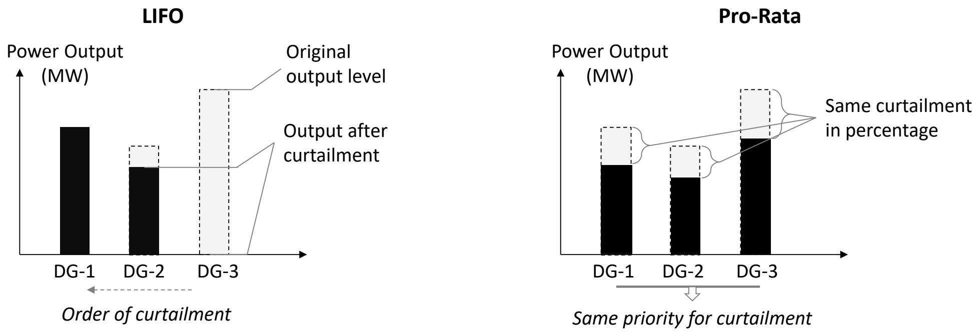
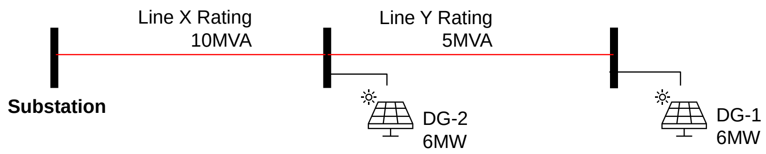


Fig. 1 Illustration of LIFO and Pro-Rata curtailment rules

Although the basic concepts of the above curtailment rules are easy to understand, how to implement such controls in the real world can be challenging. To explain, a simple circuit example is provided in Fig. 2.



Case 1

Constraint Evaluation	Curtailment (MW)		Generation (MW)	
	ΔDG-1	ΔDG-2	DG-1	DG-2
Line X	0	2	6	4
Line Y	1	0	5	4

Case 2

Constraint Evaluation	Curtailment (MW)		Generation (MW)	
	ΔDG-1	ΔDG-2	DG-1	DG-2
Line Y	1	0	5	6
Line X	0	1	5	5

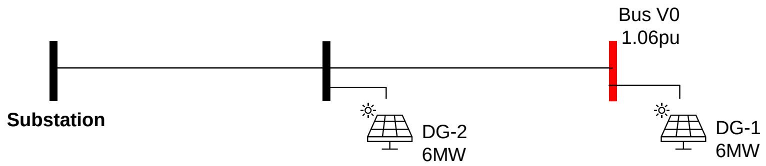
Fig. 2: An example showing the challenge of curtailment control (assuming LIFO scheme where DG-1 has higher priority than DG-2)

In this example, two DGs are connected to two different nodes in a feeder circuit. There are two line sections, X and Y, both of which are experiencing overloading if the output of the DGs are not curtailed. It is assumed that a LIFO scheme is used as the curtailment rule where DG-1 has higher priority than DG-2.

In Case 1 when the thermal constraint of Line X is first evaluated, since DG-2 needs to be curtailed before DG-1, 2MW power output is taken away from DG-2 and the total remaining power flow on Line X is reduced to 10MW. Next evaluation of the thermal constraint of Line Y requires additional 1MW curtailment from DG-1 to avoid overloading. DG-2 is not curtailed in this step because it doesn't contribute to the loading on Line Y. As the result, the total generation after curtailment is 9MW.

In Case 2 when the thermal constraint of Line Y is first evaluated, DG-1 has to be curtailed by 1MW without touching DG-2 since only DG-1 contributes to the line loading. Next evaluation of the thermal constraint of Line X curtails DG-2 by another 1MW to alleviate the overloading situation. The total generation after curtailment in this case is 10MW.

From the example above, it's clear that an optimal curtailment control can only be achieved through a properly ordered evaluation of multiple constraints. Although it's possible to develop certain rules, such as to always start from the farthest constraints towards the feeder head [3][5], to increase the chance of reaching the optimal solution in simple circuits, when the number of DGs increases and the topology of the feeder circuit becomes complicated, these rules do not provide an optimal solution. When the contribution from DGs to constraints is not either YES or NO (i.e., a binary number), but a continuous number instead, the implementation of the curtailment rules becomes even more difficult. This can be illustrated through another example as shown in Fig. 3.



Contributing Factor

$\partial V/\partial P$ (puV/MW)	
DG-1	**DG-2**
0.01	0.001

Curtailment

LIFO Priority Order	Curtailment (MW)		Generation (MW)	
	ΔDG-1	ΔDG-2	DG-1	DG-2
DG-1 > DG-2	0.4	6	5.6	0
DG-2 > DG-1	1	0	5	6

Fig. 3: Another example showing the challenge of curtailment control (assuming LIFO scheme)

Again, two DGs are connected to different nodes in a simple feeder circuit. The system constraint, however, is the voltage on the remote bus which is higher than the utility operation limit of 1.05pu before DG curtailment. The contributing factor from each DG's output power to the bus voltage is assumed to be linear and the values are provided in the table.

In the first case if we assume a LIFO scheme where DG-1 has higher priority than DG-2, since both DGs contribute to the over-voltage constraint and DG-2 should be curtailed first strictly according to the curtailment rule, all 6MW of DG-2 will be curtailed and additional 0.4MW of DG-1 must be further curtailed to bring the bus voltage down to 1.05pu. This leaves the total generation after curtailment to be 5.6MW.

In the second case if DG-2 has higher priority than DG-1, then only 1MW of power needs to be curtailed from DG-1 and the total generation after curtailment is 11MW.

The difference between the cases is significant and leads to an important question – to what extent should the curtailment rules be strictly followed? When curtailing a DG with low impact to mitigating a system violation, it may be a good idea to include the effectiveness (i.e., contributing factors) in the FICS curtailment rules through a mutually agreeable, fair measure.

Many curtailment control algorithms have been discussed in literatures. [6] discusses and compares two approaches. The first approach is modeled as a constraint satisfaction problem (CSP) by defining the DG output into multiple bandings in its variable domain. From feasible solutions that all satisfy the system operating constraints, only the one with the contractual ranking preference is selected. The second approach uses Current Tracing (CT) method to decide if a DG contributes to a constraint or not. If the answer is yes, then a pre-defined proportional sharing logic is applied to curtailing the DGs. For complicated problems where the distribution system experiences different types of constraints at many locations, the proposed methods may not work effectively.

A curtailment control algorithm based on Power Flow Sensitivity Factor (PFSF) that is calculated from the system inverse Jacobian matrix is proposed in [7]. Each DG is assigned an integer number as its priority ranking that corresponds to its PFSF and the curtailment is executed based on this number.

[8] compares three different curtailment methods: LIFO, proportional curtailment, and OPF-based optimal curtailment. In this paper, the OPF-based method is considered mutually incompatible with the other two contract-based methods.

In [9], a Quadratic Programming (QP) formulation is developed from relaxing quadratic constraints and defining the objective function as minimizing the summation of line flows. Contractual terms are not considered in this approach.

[10] also calculates the sensitivity factors of DG outputs to thermal loading and voltage magnitude based on load flow Jacobian matrix, and optimizes by minimizing a weighted summation of curtailment and power loss.

When contract-based curtailment rules are considered, all of the above-mentioned literatures either 1) follow a strictly defined proportional factor table or 2) minimize the total curtailments as a linear weighted-sum equation. However, the first method may lead to non-optimal result when different DGs contribute differently to the constraints, while the second method does not always guarantee a fair result when multiple DGs with the same priority rankings are contributing similarly to the constraints (e.g., connected at the same location). In addition, no discussed method addresses both the nonlinearity and the discontinuity of distribution load flow due to smart inverter functions (e.g., volt-var control as defined in IEEE1547-2018) or tap changer operation of voltage regulators.

In the next section, an iterative, Quadratic Programming (QP) based curtailment control is proposed as a solution.

II. Iterative QP Based FICS Curtailment Control

The vaguely defined priority order of DGs in LIFO or Pro-Rata scheme can be given more precise mathematical meanings when we formulate the problem as a Quadratic Programming (QP) problem. Equation (1) gives the canonical formula of QP.

$$\begin{gathered}\text{minimize } f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{c}^T\boldsymbol{x} \\ \textit{subject to}: \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b}\end{gathered} \tag{1}$$

where $\boldsymbol{c}$ is an N-dimensional real-valued vector, $\boldsymbol{Q}$ is an N×N dimensional real-valued matrix, $\boldsymbol{A}$ is an M×N dimensional real-valued matrix, and $\boldsymbol{b}$ is an M-dimensional real-valued vector.

By defining **x** as a vector of individual DG curtailments, we have (2) where $P_{old(i)}$ and $P_{new(i)}$ are the original and new (after curtailment) power of the ith DG respectively.

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \text{ where } x_i = P_{old(i)} - P_{new(i)} \tag{2}$$

There are three types of inequality constraints in the above-discussed examples. The first type limits the range of the DG curtailment to be non-negative and less than the original power.

$$0 \le x_i \le P_{old(i)} \tag{3}$$

The second type describes the thermal constraint as shown in (4): the power flow on any line segment must be less than its loading capability. $L_{ini(j)}$ is the initial loading level on the j^{th} line segment before any curtailment, $L_{lim(j)}$ is the loading limit of the j^{th} line segment, and $\frac{\partial L_j}{\partial x_i}$ represents the contribution factor of the i^{th} DG's curtailment to the loading on the j^{th} line segment.

$$L_j = L_{ini(j)} + \sum_{i=1}^{N} \frac{\partial L_j}{\partial x_i} x_i \le L_{lim(j)} \tag{4}$$

Similarly, the third type describes the voltage constraint as shown in (5) where $V_{ini(k)}$ is the initial voltage on bus k, V_{max} is the maximum allowable bus voltage, and $\frac{\partial V_k}{\partial x_i}$ represents the contribution factor of the i^{th} DG's curtailment to the voltage on bus k.

$$V_k = V_{ini(k)} + \sum_{i=1}^{N} \frac{\partial V_k}{\partial x_i} x_i \le V_{max} \tag{5}$$

Although the above constraint only considers the violation mitigation in the over-voltage direction, it's similarly easy to also apply to constraint in the under-voltage direction.
Combining (3)-(5) together generates (6) below.

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \\ \frac{\partial L_1}{\partial x_1} & \cdots & \frac{\partial L_1}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial L_{M1}}{\partial x_1} & \cdots & \frac{\partial L_{M1}}{\partial x_N} \\ \frac{\partial V_1}{\partial x_1} & \cdots & \frac{\partial V_1}{\partial x_N} \\ \vdots & \ddots & \vdots \\ \frac{\partial V_{M2}}{\partial x_1} & \cdots & \frac{\partial V_{M2}}{\partial x_N} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \le \begin{bmatrix} 0 \\ \vdots \\ 0 \\ P_{avl(1)} \\ \vdots \\ P_{avl(N)} \\ L_{lim(1)} - L_{ini(1)} \\ \vdots \\ L_{lim(M1)} - L_{ini(M1)} \\ V_{max} - V_{ini(1)} \\ \vdots \\ V_{max} - V_{ini(M2)} \end{bmatrix} \tag{6}$$

There are two main objectives that this QP algorithm is trying to achieve in the curtailment control: 1) to minimize the total curtailment from all DGs, 2) to fairly reflect the curtailment rule that was previously agreed between the utility and the owners. Recognizing that the second objective may take precedence in the real world as it is often contractual requirement, an objective function in (7) is proposed to the control.

$$minimize\ f(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T \begin{bmatrix} q_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & q_N \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}^T \boldsymbol{x} \tag{7}$$

This objective function minimizes the weighted summation of squared curtailments where the weighting factor, q_i, controls the priority order of each DG. At the optimal solution, the contributions from individual DGs to the objective function are the respective partial derivatives as shown in (8). For each additional unit of curtailment, the incremental cost from the i^{th} DG equals to its total curtailment scaled by q_i. The way for the objective function to reach its minimum is to always utilize the curtailment from the DG that yield the minimum marginal cost within the inequality constraints. If $q_i = 10q_j$, then the optimization algorithm will tend to curtail the j^{th} DG nine times more than the i^{th}

DG if unconstrained. Similarly, if all weighting factors are set the same, then all DGs are curtailed equally, and a Pro-Rata rule is realized.

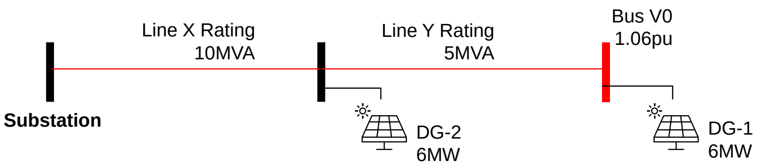
$$\frac{\partial f(\boldsymbol{x})}{\partial x_i} = q_i x_i \tag{8}$$

When applying the proposed QP algorithm to the problem discussed in Fig. 2 and Fig. 3 (by combining both the thermal and the over-voltage constraints), the following numerical math expression is resulted.

$$\begin{gathered} minimize\ f(\boldsymbol{x}) = \frac{1}{2}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ subject\ to: \\ \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ -1 & 0 \\ -0.01 & -0.001 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 0 \\ 0 \\ 6 \\ 6 \\ 10-12 \\ 5-6 \\ 1.05-1.06 \end{bmatrix} \end{gathered} \tag{9}$$

Fig. 4 summarizes the optimal solutions of four different curtailment rules. Both LIFO and Pro-Rata are now unified by simply assigning different weighting factors to individual DGs. One can also imagine the scenarios where LIFO and Pro-Rata can be combined to give different priority orders to different groups of DGs while in the same group all DGs are equally curtailed.

Note that in the four evaluated cases of Fig. 4, there are two where DG-1 has a lower priority than DG-2. By using different sets of weighting factors, such priority differentiation can be enhanced or relaxed depending on the users' choice, and the corresponding results may be different to reflect such a choice. It's also interesting to note that all the four results give the same total curtailment and final generation in this problem. The selection of weighting factors does change the individual DG curtailments but renders the same result from the utility point of view. In a different problem, however, the choice of curtailment rules may have impact to the total curtailment and the generation depending on the specific limiting constraints.



Contributing Factor

$\partial V/\partial P$ (puV/MW)	
DG-1	DG-2
0.01	0.001

Line	$\partial L/\partial P$ (MW/MW)	
	DG-1	DG-2
X	1.0	1.0
Y	1.0	0.0

Curtailment

Priority Order	Curtailment (MW)		Generation (MW)	
	ΔDG-1	ΔDG-2	DG-1	DG-2
DG-1 > DG-2 (q_1=10q_2)	1.00	1.00	5.00	5.00
DG-1 < DG-2 (q_2=10q_1)	1.82	0.18	4.18	5.82
DG-1 < DG-2 (q_2=100q_1)	1.98	0.02	4.02	5.98
DG-1 = DG-2 (Pro-Rata)	1.00	1.00	5.00	5.00

Fig. 4: Optimization solutions to the combined example in Fig. 2 and Fig. 3

III. Application to Distribution Feeder Circuits

The proposed algorithm was applied by National Grid to model an ANM application with four PV generators that are flexibly connected to different locations within a distribution network experiencing high penetration of PV generation, leading to congestion on its substation transformer. Using the control algorithm, the operation of these PV generators is simulated, and the annual curtailed energy is estimated for the next 10 years. Such information provides important insights to utilities and developers considering the economic value of FICS and ANM applications. For readers' convenience of this paper, a publicly available IEEE test feeder is used for demonstration in the next section.

In applying the algorithm to a real-world feeder system, the first challenge is to find out the sensitivity matrix **A** that describes the contribution factors of DG curtailments to system constraints. Instead of deriving the factors as analytical solutions, there are a few reasons to solve them numerically using load flow calculations on the feeder models.

- A distribution system is large-signal-nonlinear due to the nonlinear characteristics of loads (e.g., constant power or impedance loads), discrete control of voltage regulating devices (e.g., voltage regulators and controlled capacitor banks), and quadratic equations of power flow.
- The contribution factors are highly dependent on the system operating conditions and may vary in a wide range (e.g., the contribution of a DG's curtailment to a line loading may change from negative to positive depending on the direction of the power flow and the reactive power control mode of the DG)

Using load-flow-based method to linearize the studied feeder system and calculate the sensitivity matrix **A** in the vicinity of its current operating condition can avoid performing complicated analytical deviations as well as make the method easily adaptive to actual distribution operation where real time measurement feedback can help compensate the model errors.

Fig. 5 shows the flow chart that describes the implementation of the proposed curtailment control algorithm.

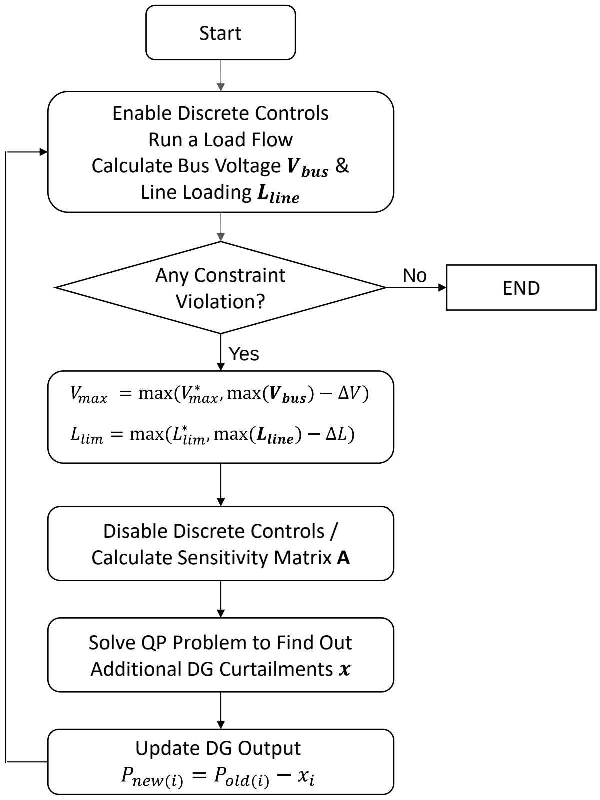


Fig. 5: Flow chart showing the implementation of the curtailment control algorithm to a realistic feeder model

Please note that this method utilizes an iterative process to determine the curtailments in one or a few steps to address the voltage and thermal constraints. For example, if the ultimate over-voltage limit V^*_{max} is 1.05pu per utility's operational requirement and in the first iteration the maximum voltage of all evaluated buses max ($\boldsymbol{V_{bus}}$) is 1.07pu, the algorithm may try to first bring the maximum voltage to below 1.06pu (i.e., ΔV=0.01pu) so the assumed system linearity is valid in close vicinity of the current operating state. How to set proper values to ΔV and ΔL is a user selection based on the studied systems. In most cases, ΔL can be as large as to bring the maximum line loading to below its final threshold in one step while ΔV is better to be no higher than 0.01pu. At the end of the iteration when all constraint violations are addressed, the final DG output shall be calculated as in (10 where $P_{avl(i)}$ is the available power of the i^{th} DG.

$$P_{curtailment\ (i)} = P_{avl(i)} - P_{new\ (i)} \tag{10}$$

Calculation of matrix **A** is relatively straightforward. A baseline load flow case is first established with all discrete controls of voltage regulators and capacitor banks enabled. By applying a small perturbation to the output of each DG and running one load flow (with all discrete controls disabled this time), the contributions of the perturbation to all bus voltages and line flows across the whole feeder (one column of **A**) can be derived. To limit the size of the matrix and improve the computational performance, only the items of those violating buses and lines need to be evaluated.

The described curtailment control logic is applied to the IEEE 8500-bus test feeder for demonstration. Six photovoltaic (PV) generators are added to randomly selected buses in the feeder circuit. Table I lists the names, locations, rated powers, and other relevant information of the added PV generators. To prevent the reverse power from the generators creating excessively high feeder voltage and making the case too unrealistic, non-unity power factor operation mode is assumed to be used by those generators connected at distant buses from the substation.

Table I: Summary of added PV generators into the IEEE 8500 bus feeder

Name	Bus	Distance from Substation (km)	Rated kW	Constant PF (- inductive)
PV-1	E192201	0.56	4000	1.0
PV-2	E183493	4.36	4000	-0.95
PV-3	M1089141	5.99	3000	-0.95
PV-4	N1134475	7.76	3000	-0.95
PV-5	M1047292	9.81	1500	-0.95
PV-6	L3254213	14.4	1500	-0.95

The resulted circuit diagram of the feeder is plotted in Fig. 6 where not only the PV generators, but also the voltage regulators and the capacitor banks are highlighted for easy reference.

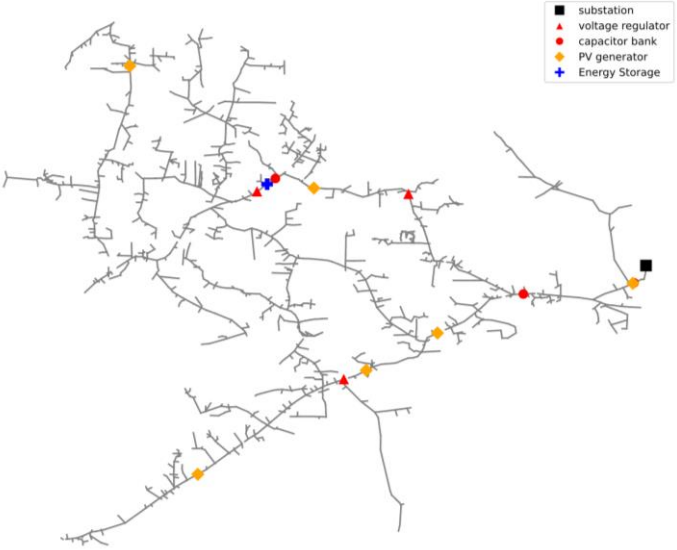


Fig. 6: Circuit diagram of the IEEE 8500-bus feeder (neglect the energy storage system labeled in this figure for now)

A full year quadratic static hourly time series simulation is performed on the modified IEEE 8500-bus feeder. For each hour of the year, both thermal and voltage violations on all main line nodes and lines are evaluated and addressed through PV power curtailment if needed. The hourly sampled load and solar generation profiles are taken from real utility historical data for a similarly sized feeder. The annual total curtailment and energy yield for each individual PV generators are summarized in Table II.

Table II: Annual curtailment summary table from the simulation

Name	Generation (MW·Hr)	Curtailment (MW·Hr)	Energy Yield (%)
PV-1	5866	268	95.6%
PV-2	5825	309	95.0%
PV-3	4324	277	94.0%
PV-4	4369	232	95.0%
PV-5	2121	179	92.2%
PV-6	2188	112	95.1%
All	24693	1378	94.7%

To provide a better visual illustration, one month of the simulation results are presented in Fig. 7. The first subplot shows the load and generation profiles used in this simulation. The second subplot compares the maximum line loading across the whole feeder before and after the curtailment. It can be noticed that during the worst hour of the month, the maximum line loading reaches 130%. With generation being curtailed, the maximum line loading is successfully maintained below 100%. Similarly, the third subplot compares the maximum bus voltage before and after the curtailments. The objective of keeping voltage always lower than 1.05pu is achieved here through curtailment. The fourth and the fifth subplots give the hourly power curtailment and the cumulative energy curtailment (of the month) of each PV generator. It must be pointed out that although all the PV generators are assigned the same priority order (i.e., Pro-Rata) in this simulation, the control algorithm calculates different energy curtailments in per-unit for them based on their different contributions to the violating bus voltage and/or line loading. This is clearly shown in the bottom two subplots in Fig. 7 and the annual energy yield column in Table II.

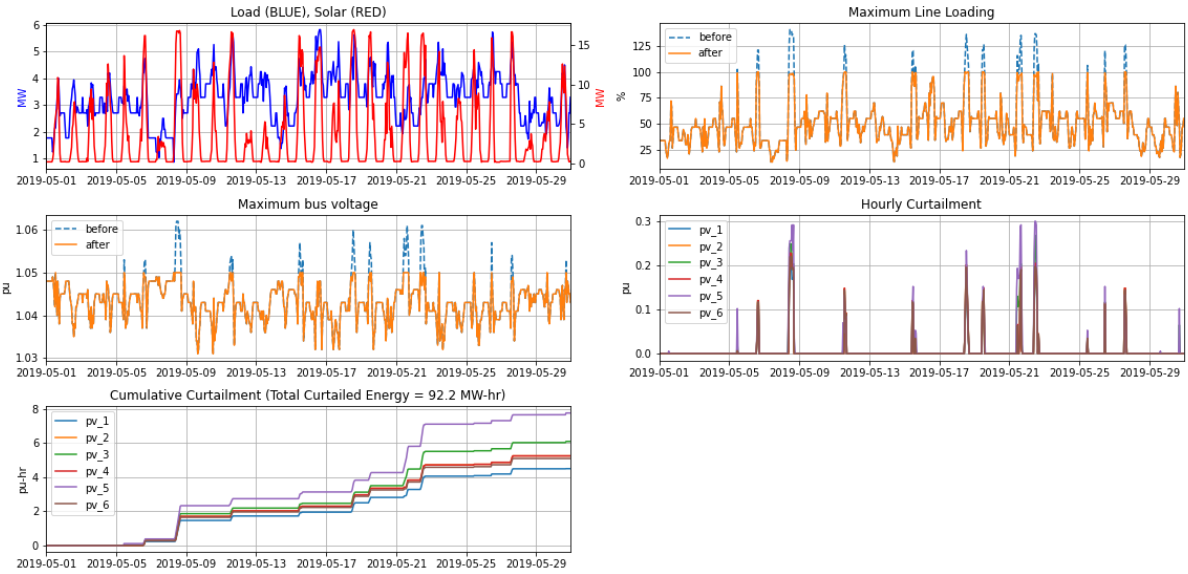


Fig. 7: Time series simulation result for one month

IV. Additional Discussion

A. Flexible Objective Setting

The proposed QP-based curtailment control method offers adequate flexibility for the users to choose other objective functions based upon specific application needs. For example, a utility may have actual $/MW cost assigned to individual DGs to compensate for the curtailment-induced revenue loss included in a contractual agreement. Using that **c** array in the objective function in (1), one can easily combine the financial objective together with priority-based curtailment rules to achieve a reasonably fair curtailment result. It's also possible to tune the parameters in the objective function on a time-variant base to readjust the preferences when certain limits (e.g., the annual curtailment energy on a certain DG) is not to be exceeded by the end of the year.

B. Thermal Constraints versus Voltage Constraints

Most FICS applications focus on resolving system thermal constraints on costly system infrastructure assets such as a substation transformer more than feeder thermal constraints or voltage constraints on the feeder, because there usually are less expensive traditional solutions that can be utilized to resolve voltage issues (e.g., adding voltage regulators or capacitor banks, adjusting control setting of existing devices, enable smart inverter functions of the DGs to regulate voltage, etc.). The proposed control algorithm applies similarly with or without the inclusion of voltage constraints. In fact, it will solve faster with less iterations in most cases if only thermal constraints are to be considered.

C. Distributed Resources with Time-Dependent Constraints

In the studied cases of DG curtailment, there is no time dependency of either the objective or the constraints. In another word, the curtailment result from one time stamp has no impact to the result in the next time stamp. However, it is possible extend the proposed algorithm to other types of distributed resources where the constraints may be time dependent. In the following part of this section, we will consider the control of a battery energy storage resource and discuss how to manage its real power output to address system operational constraints with minimal curtailments from other DGs.

For a given time stamp h, if we define the real power output of the battery system as $x_{ES(th)}$, we can write the first set of constraints in the following inequalities, where $P_{chg\_\max(th)}$ is the charging power limit and $P_{dis\_\max(th)}$ is the discharging power limit for this time stamp. Both limits may or may not be time variant.

$$-P_{chg\_\max(th)} \le x_{ES(th)} \le P_{dis\_\max(th)} \tag{11}$$

When included in the constraints of the QP algorithm, it's equivalent to add the following rows in matrix **A** and vector **b**.

$$\begin{bmatrix} & \vdots & \\ \cdots & -1. & \cdots \\ \cdots & 1.0 & \cdots \\ & \vdots & \end{bmatrix} \begin{bmatrix} \vdots \\ x_{ES(th)} \\ \vdots \end{bmatrix} \le \begin{bmatrix} \vdots \\ P_{chg\_\max(th)} \\ P_{dis\_\max(th)} \\ \vdots \end{bmatrix} \tag{12}$$

The state of charge (SOC) of the battery system at time stamp h can be calculated as in (13). $SOC_{ES(t0)}$ is the initial SOC at time zero, E_{Rate} is the rated energy capacity of the battery, and Δt is the interval step between time stamps.

$$SOC_{ES(th)} = SOC_{ES(t0)} - \frac{1}{E_{Rate}} \sum_{i=1}^{h} x_{ES(ti)} \cdot \Delta t \tag{13}$$

The SOC of a battery must be within a reasonable range depending on the product specification or additional operation requirement.

$$SOC_{ES\_MIN} \le SOC_{ES(th)} \le SOC_{ES\_MAX} \tag{14}$$

The above constraints are added to the QP formula by inserting the following rows in (15) to **A** and **b**.

$$\begin{bmatrix} \vdots \\ 1, \dots 1, \dots 0, \dots 0 \\ -1, \dots -1, \dots 0, \dots 0 \\ \vdots \end{bmatrix} \begin{bmatrix} x_{ES(t1)} \\ \vdots \\ x_{ES(th)} \\ \vdots \\ x_{ES(tk)} \\ \vdots \\ x_{ES(tZ)} \end{bmatrix} \le \begin{bmatrix} \vdots \\ \frac{E_{Rate}}{\Delta t}\left(SOC_{ES(t0)} - SOC_{ES\_MIN}\right) \\ -\frac{E_{Rate}}{\Delta t}\left(SOC_{ES(t0)} - SOC_{ES\_MAX}\right) \\ \vdots \end{bmatrix} \tag{15}$$

The time dependency of the SOC integration is now properly represented.

It must be pointed out that the above rows only reflect the SOC constraints at time stamp h. Similar inequalities need to be added for every time stamp in the studied time window from *t1* to *tZ*. In the discussed application of employing battery to reduce PV curtailment, it's reasonable to assume the operation of the battery system to have a strong diurnal pattern. In such a case, the time window for the QP problem formulation can be set to 24 hours as a nature operational period.

D. Test Case Demonstration

A 2.5MW - 2hr battery energy storage system is added to a randomly selected bus "m1069500" in the IEEE-8500 test feeder in addition to the six PV sites that are already included in the previous study in Chapter III. The location of this battery system has been labeled in Fig. 6.

Applying the described QP algorithm to this new problem with the following assumptions, an optimal PV curtailment and battery operation strategy can be found out.

- All six PV sites are curtailed on Pro-Rata rule
- The objective function is to minimize the total PV curtailment as well as the curtailed generation revenue based on a daily energy price curve show in Fig. 8
- The energy storage system needs to work between a minimum SOC of 20% and a maximum SOC of 100%

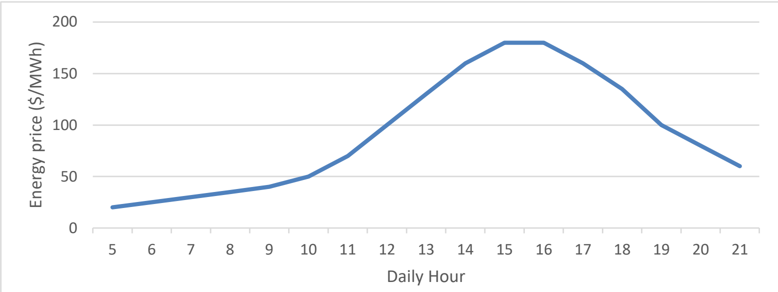


Fig. 8: An arbitrarily designed energy price curve for each simulated day

The annual curtailments of each PV site are summarized in Table III. A comparison to Table II shows that the battery storage system saves 560 MWh of curtailed energy generation. When compared with the case in Fig. 7, the increased energy sale from all PV generators is $80,782.60 using the assumed energy price in Fig. 8. It shall be noticed that the benefits introduced by the energy storage system to all PV sites are not the same. Depending on the location of the energy storage system, its power output could alleviate the system constraints for some PV sites more than the others.

Table III: Annual curtailment summary table from the new simulation

Name	Generation (MW·Hr)	Curtailment (MW·Hr)	Energy Yield (%)	Reduced Curtailment (MW*hr)
PV-1	6088	47	99.2%	221
PV-2	5933	201	96.7%	108
PV-3	4411	189	95.9%	88
PV-4	4455	145	96.8%	86
PV-5	2113	187	91.9%	-8
PV-6	2252	48	97.9%	64
All	25253	818	96.9%	560

Fig. 9 plots the simulation result for one heavily curtailed day in May 2019. The battery system charges at the hours with the highest energy price to both reduce PV curtailment and maximize generation revenue until the SOC reaches 100%. Because there is more curtailed energy than what the battery can absorb before it's fully charged, it's expected that other charging profiles during the day would result in the same amount of generation saving. However, the shape of the energy priced is an additional impact factor to the final result.

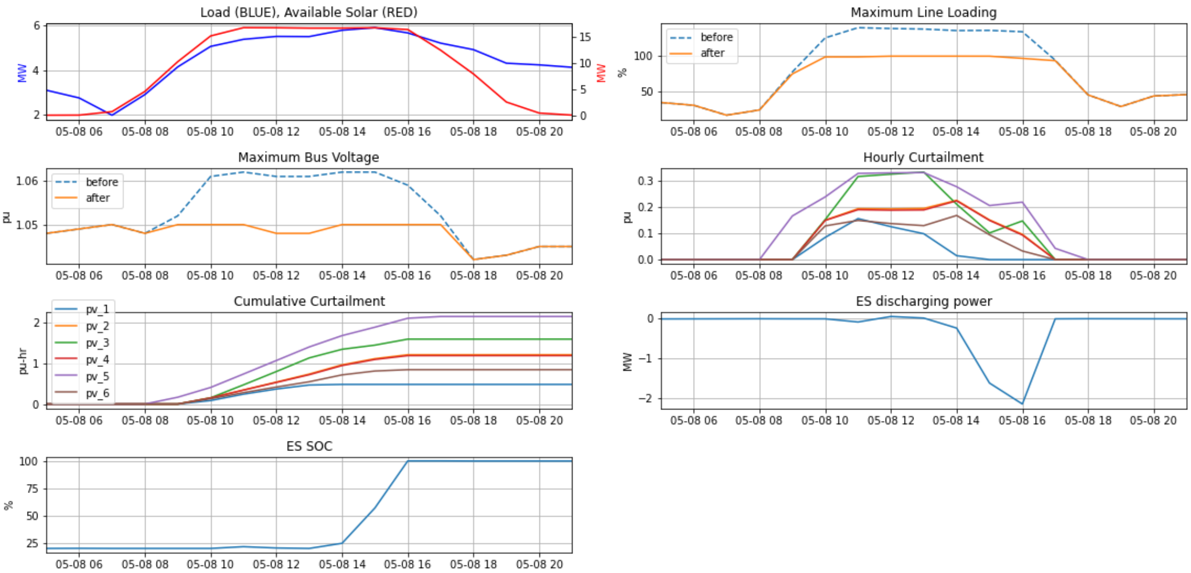


Fig. 9: Time series simulation result of the energy storage case for one day

V. CONCLUSIONS

This paper proposes a QP-based method to optimally manage the active power output of multiple DG units that are flexibly interconnected to a distribution system. Hourly simulation of a decent size feeder circuit for a full year proves the solution to be both computational efficient and easy-to-configure to address various customer-specific objectives, operational rules, and complex system constraints. Taking the IEEE 8500-bus feeder circuit as an example, to complete the annual simulation (as its result shown in Fig. 7) with hourly data takes 143 minutes on a laptop with Windows 10, Intel i7-9850H 2.6GHz CPU, and 16GB memory. This is equivalent to an average computation time of <1s per time stamp. The simulation is performed using OpenDSS [11] and the QP solver used is an open-source Python package named "CVXOPT" [12]. Further, by using the time series acceleration method as described in [13], the total computation time for the annual analysis can be reduced to <4 minutes hence make the proposed method manageable and practical.

Extension of the algorithm to support the operation of energy storage systems further illustrates the capability of the proposed method in dealing with time dependent constraints and the flexibility for achieving various objective functions.

A real utility's distribution management system (DMS) or distributed energy resource management system (DERMS) may not resemble the proposed method and may control the DGs in a very different way to accommodate non-ideal conditions such as latency in system telemetry and challenges with conducting load flows in near real-time. As a result, actual curtailment experienced by a DG under a FICS scheme may differ from what is modeled through simulation. However, one can still utilize the method to render useful book-end insights for system planning and/or project screening.

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