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Quantum-Assisted Contingency Analysis – Developing a Software-as-a-Service (SaaS) Tool to Reinforce the Cleantech in the Face of Climate Change

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SUMMARY

Climate change and related natural disasters have become more disruptive in the U.S. and worldwide, resulting in substantial momentum to increase the capacity and generation of renewable energy sources and reaching net-zero emissions. However, intermittent renewable energy sources, along with fast changing customer load and behavior, will considerably impact the electric power grid. Significant investments on smart grid technology over the past two decades have resulted in advanced grid monitoring and measurement, resulting in the continual collection of myriads of data from various parts of the grid. Computing tools have accordingly resurfaced as enabling means that can convert data to information and ultimately to actions, which will further play a more significant role as the grid is being upgraded over the next several decades. These emerging changes raise substantial challenges for system operators, especially in grid coordination and management. Grid operators call for new analytics and computing methods to develop sustained innovation for the decision-making, planning, and operation of the fast-evolving power grid. The objective of this paper is to provide an innovative contingency analysis tool capable of increasing the grid reliability, resiliency, and sustainability in the face of climate change. This is carried out by leveraging quantum computing and developing a software platform that can potentially offer billions of dollars in annual savings for grid operators in avoided costs.

KEYWORDS

Quantum Computing, Contingency analysis, Resiliency, Climate change

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1. INTRODUCTION

Power Flow is the keystone of electric utilities' decision-making in grid operation, control, and planning. The power flow solution is the amount of electricity flow for each line in transmission and distribution grids, along with nodal voltage values. Power flow has a multitude of applications, including but not limited to resource management, portfolio optimization, security analysis and capacity planning. It is a challenging task to solve this problem accurately and efficiently due to the nonlinearity induced by the laws of physics. However, a fast and accurate power flow solution is more important than ever considering the growing need for security and resilience against natural disasters, and increasing customer expectations of high reliability and quality power.

Grid operators need to operate the grid with no-limit violations even when subject to regular contingencies such as faults and other disturbances. The standard operating paradigm is to be at least N−1 reliable, meaning that there would be no limit violations if any credible contingency were to occur. N−1 reliability is assessed using contingency analysis, which in its simplest form consists of running potentially thousands of power flow solutions, each considering a different contingency. Online contingency analysis is commonly run in electric control centers for about a 5-minute interval. Since each contingency is independent, contingency analysis may be easily parallelized. However, considering the potentially large number of contingencies, especially in case of higher contingency orders such as N-2 or N-3, even parallelization may fail to investigate all potential scenarios in a reasonable amount of time.

The traditional methods to solve contingency analysis involved reformulation, approximations, or parallel processing. These methods may emerge ineffective to smart grids problems due to a lack of mathematical convergence guarantee and unscalable analytics. To address these challenges, there is a significant need for new analytics and computation to speed up decision-making, planning, and control of the system to sub-seconds while ensuring to build reliability, resiliency, and sustainability in the grid of the future. In this paper, we propose a contingency analysis model for a quantum Software-as-a-Service (SaaS) tool, to reinforce cleantech within power grids. Quantum computers can offer immense processing power through simultaneous multiple-function execution [1][2], and this characteristic will be used to our advantage in the proposed model.

2. QUANTUM ALGORITHM FOR GRID CONTINGENCY ANALYSIS

Grid analytics signifies a set of *Software-as-a-Service* (SaaS) tools to support accurate modeling appropriate for various time scales of decision-making at different layers of the complex interconnected power grid. Given the growing complexity, along with the traditional methods of computing, innovative solutions need to be offered to address the new set of challenges. A viable and emerging solution is the Quantum Computing technology. This paper focuses on the core of today's energy technology movement by developing a quantum solution to one of the most fundamental power system problems, i.e., *contingency analysis,* and builds the foundation of a Software-as-a-Service (SaaS) tool called Quantum Contingency Analysis (QCA).

Classical computers work by converting information to a series of binary digits, or bits, and operating on these bits using integrated circuits (ICs) containing billions of transistors. There are only two feasible values for each bit, 0 or 1. Computers process the data by manipulating these bits. A quantum computer also reflects data as a series of bits, called qubits. Like a normal bit, a qubit could be either 0 or 1, but unlike a regular bit, a qubit may also be simultaneously in both states. This ability to simultaneously occupy all possible binary states results in a potentially exponential scaling advantage in the number of qubits in a system.

Based on the characteristics of a qubit, with N qubits, a total of 2^N numbers can be operated simultaneously. This property is called superposition and denotes that a computer that uses this property can process data much faster while using less energy. There are many power system problems that require significant processing capabilities to address size and scalability issues. Some of these issues are addressed by reformulation, approximations, or parallel processing. However, the growing size and needs require a solution from the exact system (without any approximations) in a timely manner. Contingency analysis is a great example of such problems.

The power system contingency analysis represents the system ability to successfully handle limited unexpected events, such as the effects of component failures. Contingency analysis is commonly evaluated by considering probable component failures and calculating resulting overloads on other parts of the grid. Contingency considerations enable grid operators to better prepare for and react to outages through carefully planned preventive and corrective actions [4]. Contingency analysis may require solving potentially thousands of power flow scenarios, each considering a different contingency, and accordingly assessing the state of the system. The existing security analysis methods merely focus on the outage of a limited number of grid components, typically one (the common N-1 reliability criterion). Ongoing research focuses on developing new uncertainty-based and statistical methods to provide probabilistic solutions accounting for uncertainty.

One major reason to look into probabilistic methods is the lack of adequate computational capabilities to perform higher-order deterministic studies, i.e., N-m. This need is heightened due to the growing number of natural disasters that result in a simultaneous outage of several grid components and thus call for much more complex studies. In contingency analysis the state of the system in response to any potential contingency, i.e., the outage of units and/or lines, is evaluated. We base our model on the Harrow-Hassidim-Lloyd (HHL) quantum algorithm. This algorithm offers a proven speedup over classical algorithms in solving a system of linear equations. The IEEE 300-bus test system is used for the studies and to investigate the potential computational speedup over classical methods for solving N-m contingency analysis.

3. QUANTUM ALGORITHM

Quantum computers are proven to provide an exponential speedup for solving a system of linear equations (SLE) [3][4], which are common in many science and engineering problems. The power flow analysis, and accorindgly the contingency analysis, is an SLE that can be efficiently solved using a quantum computer. We define power flow using an admittance matrix $B \in \mathbb{R}^{\mathrm{N\times N}}$ and a nodal net injection vector $p \in \mathbb{R}^N$, with the objective of finding nodal voltage angles $\theta \in \mathbb{R}^N$, where $B\theta = p$.

The admittance matrix has at most s nonzero entries per row or column, so the power flow problem is s-sparse, requiring $O(N\ s\kappa\ \log(1/\mathcal{E}))$ running time using the conjugate gradient method in a classical computer [3]. N is the system size. Using the HHL quantum algorithm, this running time is revised to $O(\log{(N)}\ s^2k^2/\mathcal{E})$ [5]. Here k denotes the system condition number and $\mathcal{E}$ represents the accuracy of the approximation. To achieve the HHL model of the power flow problem, the power flow equations are first rescaled/normalized. This is carried out by normalizing p and mapping to the respective quantum state $|p\rangle$, and similarly by normalizin θ and mapping to the respective quantum states $|\theta\rangle$.

In the HHL algorithm, matrix B should be Hermitian, i.e., its conjugate transpose is similar to itself. This is true for the power flow model. For the AC power flow model, the B matrix needs to be extended to make it Hermitian, which is a straightforward task. The HHL algorithm is implemented as shown in Figure 1. Three quantum registers are used to store a binary representation of the eigenvalues of b, represent the solution vector, and present ancilla qubits. These registers are set to $|0\rangle$ at the beginning of each computation.

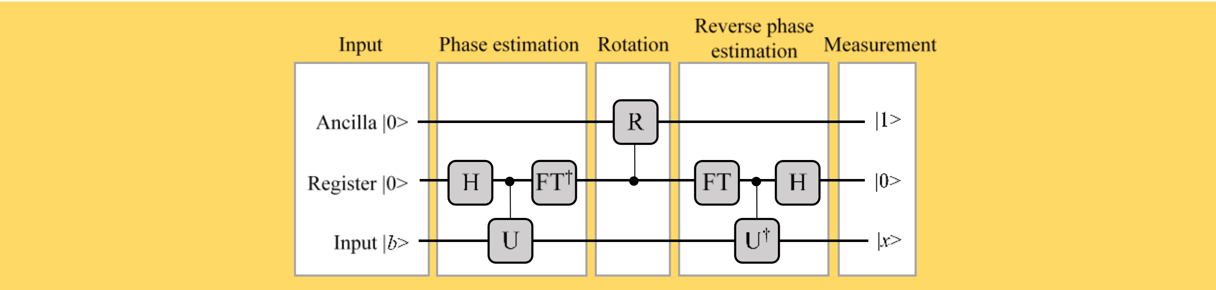

Figure 1. HHL algorithm process

To start the process, we initially load the data into $|p\rangle$, through the following transformation:

$$B|\theta\rangle = |p\rangle \tag{1}$$

Once loaded, we will apply a Quantum Phase Estimation (QPE) to find the eigenvector of B. This is a controlled unitary with a change of basis that maps the eigenvalues onto the working memory. We will define QPE with $U = e^{iBt}$, where

$$e^{iBt} := \sum_j e^{i\lambda_j t} |u_j\rangle\langle u_j|, \tag{2}$$

Then, for the eigenvector $|u_j\rangle_\beta$, which has eigenvalue $e^{i\lambda jt}$, QPE will output $|\tilde{\lambda}_i\rangle_\alpha |u_i\rangle_\beta$, where $\tilde{\lambda}_i$ represents an α-bit binary approximation to $2^\alpha \lambda_j t/2\pi$. Therefore, if each λ_j can be correctly represented with α bits,

$$QPE(e^{iB2\pi} \sum_j p_j |0\rangle_\alpha |u_j\rangle_\beta) = \sum_j p_j |\lambda_j\rangle\langle u_j| \tag{3}$$

The quantum state of the register in the eigenbasis of b is:

$$\sum_j p_j |\lambda_j\rangle_\alpha |u_j\rangle_\alpha \tag{4}$$

where $|\lambda_j\rangle_\alpha$ is the α-bit binary representation of λ_j. We now add the ancilla qubit and apply a controlled rotation conditioned on $|\lambda_j\rangle$.

2) Controlled Rotation. An ancilla qubit is added and applies a rotation conditioned on $|\lambda_j\rangle$:

$$\sum_j p_j |\lambda_j\rangle_\alpha |u_j\rangle_\alpha \left(\sqrt{1 - \frac{C^2}{{\lambda_j}^2}} |0\rangle + \frac{C}{\lambda_j} |1\rangle \right) \quad (5)$$

where *C* is a normalization constant. This step is followed by a reverse QPE.

3) Uncomputation. This step applies a reverse QPE to uncompute the results of the previous step:

$$\sum_j p_j |0\rangle_\alpha |u_j\rangle_{\alpha\,,} \left(\sqrt{1 - \frac{C^2}{{\lambda_j}^2}} |0\rangle + \frac{C}{\lambda_j} |1\rangle \right) \quad (6)$$

4) Measurement. This step measures the ancilla qubit, and providing the following post-measurement state when the outcome is 1:

$$\left(\sqrt{\frac{1}{\sum_j |p_j|^2 / |\lambda_j|^2}} \right) \sum_j p_j |0\rangle_\alpha |u_j\rangle_\alpha \quad (7)$$

4. PROOF-OF-CONCEPT STUDIES

A three-node system is used for studying how the HHL algorithm can be employed to solve the contingency analysis problem. This system has three buses and three lines. The generation from units at nodes 1 and 2 are at 20 MW and 60 MW, respectively. There is a load of 80 MW at node 3. The admittance matrix is not invertible, so we assume that the voltage angle at node 1 has a value of 0, i.e., $\theta_1 = 0$. This is a realistic assumption as there is always at least one node that acts as the slack bus and provides a reference for nodal voltage angles. By writing the power flow equations and factoring out a constant from both sides, the system of linear equations will be as follows:

$$\begin{bmatrix} 1 & -0.2 \\ -0.2 & 1 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0.6 \\ -0.8 \end{bmatrix} \quad (8)$$

Consider β=1 qubit to represent $|p\rangle$ and the solution $|\theta\rangle$, α=2 qubits to store the binary representation of the eigenvalues, and 1 ancilla qubit to save whether the conditioned rotation, hence the algorithm, is successful. For the illustrative purpose of the algorithm, a parameter *t* is selected to rescale the eigenvalues of the matrix *B* during QPE and to obtain an accurate binary representation of the rescaled eigenvalues in the α-register. In the original HHL algorithm, parameter *t* is represented as the simulation-time for the QPE part. A calculation of the eigenvalues will give λ_1= 0.8 and λ_2=1.2, and the eigenvectors $|u\rangle_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $|u\rangle_2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$, such that B $= \begin{bmatrix} 1 & -0.2 \\ -0.2 & 1 \end{bmatrix} = \sum_j \lambda_j |u\rangle_j \langle u|_j$.

The QPE will output a 2-bit binary approximation to $\frac{\lambda_i t}{2\pi}$. If we set $t = 2\pi . \frac{5}{8}$, then QPE will give a 2-bit binary, where $\frac{\lambda_1 t}{2\pi} = \frac{1}{2}$ and $\frac{\lambda_2 t}{2\pi} = \frac{3}{4}$, which are designated as $|01\rangle_\alpha$ and $|10\rangle_\alpha$. $|p\rangle$ can be accordingly written in the eigenbasis of B as $|p\rangle_\beta = 0.6|0\rangle - 0.8|1\rangle = -0.1\sqrt{2}|u_1\rangle + 0.7\sqrt{2}\,|u_2\rangle$. With this initialization, the different steps of the HHL algorithm can be applied. Applying QPE will yield $-0.1\sqrt{2}|01\rangle|u_1\rangle + 0.7\sqrt{2}|10\rangle|u_2\rangle$

The eigenvalues will give conditioned rotation with C=$\frac{5}{8} = 0.625$ to compensate from having rescaled:

$$-0.1\sqrt{2}|01\rangle|u_1\rangle\left(\sqrt{1-\frac{(0.625)^2}{(0.8)^2}}|0\rangle + \frac{0.625}{0.8}|1\rangle\right) + 0.7\sqrt{2}|10\rangle|u_2\rangle\left(\sqrt{1-\frac{(0.625)^2}{(1.2)^2}}|0\rangle + \frac{0.625}{1.2}|1\rangle\right) = -0.1\sqrt{2}|01\rangle|u_1\rangle\left(\sqrt{1-(0.781)^2}|0\rangle + 0.781|1\rangle\right) 0.7\sqrt{2}|10\rangle|u_2\rangle\left(\sqrt{1-(0.521)^2}|0\rangle + 0.521\,|1\rangle\right) \quad (9)$$

After applying reverse QPE, from (18), the qubits are in the state, up to a normalization factor N:

$$-0.1\sqrt{2}|00\rangle|u_1\rangle\left(\sqrt{1-(0.781)^2}|0\rangle + 0.781\,|1\rangle\right) + 0.7\sqrt{2}|00\rangle|u_2\rangle\left(\sqrt{1-(0.521)^2}|0\rangle + 0.521\,|1\rangle\right)$$

On the outcome of 1 when measuring the ancilla qubit, the normalized qubit state in the readout register is: $\frac{1}{N}\left(-0.1\sqrt{2}|00\rangle|u_1\rangle(0.781) + 0.7\sqrt{2}|00\rangle|u_2\rangle(0.521)\right)$, where $\frac{1}{N}\left(-0.1\sqrt{2}|u_1\rangle(0.781) + 0.7\sqrt{2}|u_2\rangle(0.521)\right) = \frac{|\theta\rangle}{||\theta||}$. with normalization constant $N = \sqrt{(0.1)^2(\sqrt{2})^2(0.781)^2 + (0.7)^2(\sqrt{2})^2(0.521)^2}$, and the state $|\theta\rangle = B^{-1}|p\rangle_\beta$.
Without using extra gates, the norm of the state $|\theta\rangle$ is computed as follows: $P[|\text{ancilla}\rangle = |1\rangle] = N^2 = ||\theta||^2$ which is the probability of measuring $|1\rangle$ in the ancilla qubit from the previous step. In the readout register a normalized quantum state $|\Psi\rangle = \frac{|\theta\rangle}{||\theta||}$ is obtained, which provides an estimate of $||\theta||$ by measuring the ancilla qubit. The estimate of $||\theta||$ is useful because in the end the non-normalized state $|\theta\rangle$ is dealt with for most of the calculations.

The problem is solved analytically in the previous section, and the result of HHL on a quantum simulator is illustrated here. First, a general HHL algorithm provided by Qiskit Aqua is run to obtain an estimate of the fidelity and other parameters. In the power flow problem, the time of the Hamiltonian simulation is set to $2\pi.\frac{5}{8}$. If the matrix has several structures, it might be possible to obtain information about the eigenvalues and use it to choose a suitable *t* and improve the accuracy of the solution returned by the HHL.

The reason for selecting *t* is to rescale the eigenvalues during QPE, which could be represented accurately with two binary digits. The fidelity of the solution should be near 1 when it is done correctly. Table 1 demonstrates the results of the studied power flow problem with different selecting *t* to rescale the eigenvalues.

Table 1. Power Flow Results

Results		
$2\pi.\frac{1}{8}$	Fidelity	**0.73591**
	Probability	0.00986
	Classical Solution	[0.4583 -0.7083]
	Quantum Solution	[0.2771-0.j -0.2918+0.j]
$2\pi.\frac{5}{8}$	Fidelity	**0.999945**
	Probability	0.020986
	Classical Solution	[0.4583 -0.7083]
	Quantum Solution	[0.4534-0.j -0.7123+0.j]
$2\pi.\frac{5}{4}$	Fidelity	**0.861293**
	Probability	0.01756
	Classical Solution	[0.4583 -0.7083]
	Quantum Solution	[0.2845-0.j -0.3759+0.j]
$2\pi.\frac{15}{8}$	Fidelity	**0.602983**
	Probability	0.01019
	Classical Solution	[0.3894 -0.8340]
	Quantum Solution	[0.1141-0.j -0.4895+0.j]
$2\pi.\frac{5}{2}$	Fidelity	Error
	Probability	Error
	Classical Solution	[0.4583 -0.7083]
	Quantum Solution	Error

The high fidelity of the results shows the promise in finding accurate solutions. It should not come as a surprise that classical solution is exact because all the default methods used are exact. Table 2 shows the characteristics of the quantum circuit used to solve this problem.

Table 2. Characteristics of the Quantum Circuit

Results		
$2\pi.\frac{1}{8}$	Circuit Width	Error
	Circuit Depth	Error
	CNOT Gates	Error
$2\pi.\frac{5}{8}$	Circuit Width	7
	Circuit Depth	102
	CNOT Gates	54
$2\pi.\frac{5}{4}$	Circuit Width	7
	Circuit Depth	126
	CNOT Gates	63
$2\pi.\frac{15}{8}$	Circuit Width	9
	Circuit Depth	134
	CNOT Gates	80
$2\pi.\frac{5}{2}$	Circuit Width	14
	Circuit Depth	165
	CNOT Gates	Error

The depth is the maximum number of gates applied to a single qubit. The width is the number of qubits required. The number of CNOTs is also provided as this number, and the width, provide a sense of whether running the circuit on current practical hardware is feasible.

In Tables 1 and 2, it is shown that how an optimal phase estimation can effect on finding the optimal circuit size as well as the accuracy in our problem. As it is shown, the optimal size of circuit is related to the optimal size of phase estimation it can gradually increase while the number phase estimation increase till an error occurs and conversely.

5. NUMERICAL SIMULATIONS

The IEEE 118-bus test system is used for solving the contingency analysis using the HHL algorithm. The quantum circuit was designed with Qiskit [6], which is an open-source software development kit for circuits level quantum computing. The circuit is simulated on the IBMQ Experience cloud quantum computing platform. The theoretical studies suggest that a quantum computer could provide higher processing power and solve this fundamental grid problem significantly faster than a classical computer for large scenarios.

The IEEE 118-bus system has 118 buses, 54 generators, and 186 transmission lines. For an N-1 study, the set of power flow equations should be solved 240 times (i.e., 54+186). For an N-2 study, this number goes up to 28,000, and for N-3 it will be close to 2.2 million. If we assume each run takes an average of 100 ms, it would take close to 2.5 days to run simulations for all contingency cases under a N-3 scenario. For higher contingency scenarios or larger systems, even the strongest classical computers may fail to perform this simulation in a timely manner. Considering k as the number of unknowns, classical computers solve linear system of equations in a timescale of order k while the proposed quantum algorithm is proved to find the solution in a timescale of order $\log(k)$.

Based on this speed up, the N-3 study can be performed in around 71 s instead of 2.5 days as in a classical computer. Therefore, employing the HHL algorithm in a quantum computer can confer an exponential speedup over the best classical algorithm. It is worth noting that the line outage distribution factor (LODF) method can also be used instead of solving power flow equations. This method, although faster than solving a linear system of equations in a classical computer, will still show scalability issues, i.e., an exponential computation time increase, when used for larger systems and higher number of contingency scenarios. Table 3 demonstrates the results of the studied contingency analysis.

Table 3. Computation time

Contingency type	**N-1**	**N-2**	**N-3**
# of times to solve power flow	240	~28,000	~2.2 million
Computation time (classical)	24 s	45 min	2.5 days
Computation time (quantum)	24 s	42 s	71 s

6. CONCLUSION

The electric power grid is facing unprecedented challenges and is undergoing major transformation, making enhanced analytics and computation of paramount importance. This paper presented an experimental quantum computing solution to the contingency analysis problem. There are two aspects that drive this emerging technology: the need for faster and more accurate decision-making tools by grid operators, and the viability of a quantum solution. Quantum computing, as discussed in this paper, represents a significant technology that can help make power systems more reliable, resilient, and sustainable. Our model helps to make effective use of large data integration of renewable energy resources, as well as enhancing the reliability by making effective use of large data integration of renewable energy resources and leads to operation cost reduction by providing faster control and decision reduce the electricity cost for both utilities and costumers.

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