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CIGRE US National Committee
2022 Grid of the Future Symposium

A Robust Energy System Under Uncertainty: Multi-level Optimization-based Frameworks

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SUMMARY

The aim is to identify the vulnerabilities of an electrical energy system to ensure its robustness and resilience. Like any other critical infrastructure (CI), power systems are subject to disruptions, that may have a significant impact on their performance. "Vulnerability analysis" in power systems is important to determine how vulnerable a system is and to detect and rank the most critical elements of a power system under a variety of high-impact low-frequency (HILF) events such as natural hazards. In this paper, first, a hierarchical leader-follower (bilevel) optimization problem is developed where the upper level (leader) tries to maximize the damage, and the lower level (follower) tries to minimize the probable consequences. Thanks to this rational strategy, the critical components whose failures lead to the largest system loss can be found.

Afterward, the proposed model is extended to immunize the system against the worst uncertainty. The proposed model is applied to the IEEE test systems. The results show that the proposed model is much more efficient than the previously reported one, where the approximated power flow equations are used in the lower level. Also, to guarantee the operational security of power systems with uncertainties, an adaptive robust trilevel optimization model for immunizing the system against the worst uncertainty has been carried out. The final one-level model has been applied to the IEEE network. Simulation results show that the power system vulnerability assessment without considering uncertainties leads to optimistic results.

KEYWORDS

Vulnerability assessment, electric power system, multi-level optimization problem, optimal power flow.

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1- Introduction

A secure, competitive and decarbonized energy system is the main goal of the energy sector's decision-makers [1, 2]. The power system, which will undergo a major restructuring in the coming years, plays a key role as a CI. Its security against various hazards and threats such as natural hazards, intentional attacks, and random failures [3] has become a growing concern. Even in a system with a low probability of occurrence of a blackout, the risk remains high as the impact of an event involves substantial social and economic costs. For instance, power failures have recently affected hundreds of millions of people. In 2003, three independent events in Iran, North America, and Italy hit 128 million people. In 2012 and 2015, 670 million Indian people and 70 million Turkish people respectively were temporarily deprived of power [4-7]. In the USA, the annual cost of weather-related blackouts ranges from $20 to $55 billion. Furthermore, global warming is worsening the events' intensity and making extreme weather much more frequent [8, 9]. Deploying robust and resilient CIs can limit these costs and control their consequences [10]. Hence, to improve the security, robustness and managing resilience, operators (decision makers) must detect the most vulnerable elements and deploy preventive and corrective actions under a variety of stochastic nature of extreme events, attack scenarios, and power systems uncertainties [11].

Scientists have been developing innovative methods to find critical components whose failures lead to the largest system loss. Generally, there are two different lines of work in the literature. Some literature work on the low-order contingencies including a single or a small number of component failures that have a very high occurrence probability. For instance, the *N*-1 security constraint that all regulatory agencies in the world require system operators to satisfy by strict security standards. Within the N-1 constraint, the system should normally continue to work after any single failure [12]. Analyzing the loss of two elements consecutively (*N*-1-1) is another example of this category [13-15]. It should be noted that the reliability concept that defines the ability of the electric power system to meet the demand with continuity and an acceptable level of quality comes under the low-order contingencies (i.e. N-k (small k) contingencies) [16]. The second line of works presents not only the low-order but also the high-order contingencies. The high-order contingencies include multiple component failures that have a very low occurrence probability but high consequences. The focus of this research proposal is on this second type of assessment *i.e.* the vulnerability analysis and resilience management of power systems to HILF [17].

The works, in this area, can range from analytical approaches such as complex network (CN) [3, 18, 19], flow-based, logical, and functional methods, to MC simulations. A detailed comparison of these approaches was recently conducted in [20] (and the references therein). Conventional CN analysis ignores the physical properties, electrical characteristics, and operational limits of power grids. It limits the scope of the analysis [21-23]. Scholars have updated the "pure" centralities to take these "extended" characteristics into account. Some studies consider the physical resistance and impedance of lines and cables as the weight on the edges [3, 24]. Complex network methods originally ignore the physics of the power system operation, an issue that was partially overcome with extended CN analysis. Nevertheless, power flow-based methods intrinsically consider physical features [25, 26]. They simulate power flows in power system planning and operations [27] with deterministic and probabilistic approaches.

Among the state-of-the-art approaches for vulnerability assessment and resilience analysis, optimization-based approaches lead to promising results without the need to rank the sets of critical assets. The application of optimization-based approaches to power system operation problems is increasing considerably [20]. For instance, a defender–attacker–defender (DAD) model is proposed to deal with the power grid protection problem under uncertain attacks using the analytic hierarchy process [28]. In [29], a unidirectional optimization model is proposed to identify and evaluate the vulnerability of integrated energy system by considering the topological and functional characteristics of the system. It shows that the pure topological metrics can result in misleading solutions. A DAD formulation is proposed to identify and protect vulnerable components of IEGS against intentional attacks in [30]. In [31], a tri-level resilient operational framework of power systems is developed, that considers firm gas supply and gas reserve contracts with gas systems in both the pre- and post-contingency stages. A security-constrained optimal power and gas flow is developed using linear sensitivity factors in [32, 33]. The contributions of the proposed models in these papers are as follows:

(1) A novel deterministic multi-period AC-based one-level mixed integer linear program (MILP) formulation of a bilevel mixed integer nonlinear programming (MINLP) problem is introduced to assess the N-k contingency analysis. An LP is proposed for AC optimal power flow (OPF) where the technical details of AC networks such as resistance, reactance, shunt susceptance, active and reactive power are properly modeled. Due to AC power flow details in the LP model, the results are more practical and useful for the system operators as compared to the models where the naïve DC OPF approximation is used. The model considers a real picture of the power grid parameters i.e., both active and reactive powers, losses, and voltage profile.
(2) An adaptive robust trilevel optimization model is introduced to assess the vulnerability of power systems. The proposed optimization model is robust against all possible realizations of uncertain load demands. Moreover, the level of robustness is controlled using predefined budget constraints.

The rest of this paper is organized as follows. Section 2 introduces assumptions, the two-stage adaptive robust vulnerability assessment, and the uncertainty set. The formulation of the proposed optimization model is presented in Appendix 2. Section 3 proposes the mathematical techniques to transform the original mixed integer tri-level nonlinear programming (MITNLP) model to an equivalent MILP model. Sections 4 presents the test cases and the numerical results. Section 5 presents concluding remarks .

2- Problem formulation

In this Section, the mathematical formulation of the AC optimal power flow (OPF)-based attacker-defender problem and proposed adaptive robust optimization (ARO)-based vulnerability assessment for power are presented. These formulations are based on the commonly used assumptions in the literature for vulnerability assessment of a power system [34-39].

In this context, according to Figure 1(A) [40, 41], the upper level models the attacker, and the lower level models the defender. The attacker as the leader starts the leader-follower game with limited disruptive resources. The defender as the follower reacts against the set of out-of-service assets to mitigate its adverse consequences. This leader-follower interaction between the attacker and the defender is modeled as the bilevel optimization problem as follows (the terms are defined in the appendix):

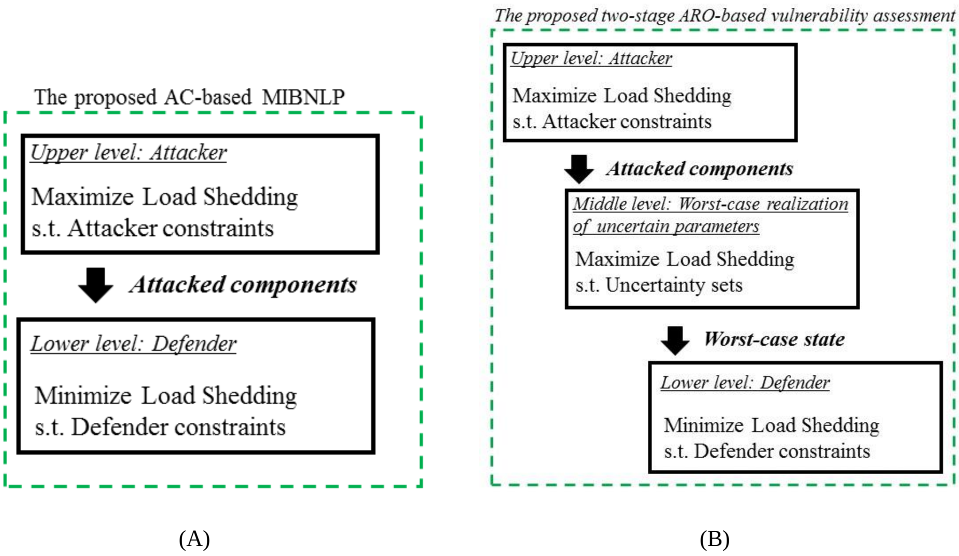

Figure 1. (A) The attacker-defender model (B) the proposed ARO-based vulnerability assessment for power systems.

$$\underset{Z}{Max}\sum_{i\in N} Ls_i^* \quad (1)$$

s.t. $$0.5\times\sum_{i,j\in N}(1-z_{ij})=NPO;\quad z_{ij}\in\{0,1\}\quad \forall i,j\in N \quad (2)$$

$$z_{ij}=z_{ji};\quad z_{ij}\in\{0,1\}\quad \forall i,j\in N \quad (3)$$

where: $$\sum_{i\in N} Ls_i^* \in \arg\left\{\underset{\substack{Pg,Qg,Ls,Lsq,\\ P,Q,V,\theta}}{Min}\sum_{i\in N} Ls_i\right\} \quad (4)$$

s.t. *AC OPF (or DC OPF)* (5)

Equations (1)-(5) of the optimization problem form a mixed-integer bilevel nonlinear program (MIBNLP) which is non-convex and NP-hard. Equations (1)-(3) model the attacker optimization problem which maximizes the load shedding subject to the limited number of plausible outages considered in constraint (2). Since $z_{ij}=z_{ji}$, (i.e. mathematically when line ij is off, line ji is automatically off), the factor 0.5 is multiplied by the total number of line outages to avoid double consideration in this formulation. If z_{ij} is 0, the line *ij* is under attack, otherwise it is safe. In the lower-level defender problem (4)-(5), unlike the previous approaches [34-39], the AC OPF is used as the defender tool to mitigate the adverse consequences of the outages which is introduced in Appendix 2. Equation (4) is the objective function of the defender to minimize the damage. The asterisks in (1) and (4) emphasizes that Ls_i is decided in the lower-level problem.

To consider the uncertainty, according to Figure 1(B) [42, 43], a set of first-stage decisions are made before the realization of uncertainty (attacker's decisions) and a set of second-stage decisions are made once the uncertain parameters are revealed (defender's decision). So, the proposed model is *fully adaptive* to the specific realization of uncertainties. This two-stage model includes three levels:

1. The upper level models the attacker as the leader prior to the uncertainty realization. The attacker maximizes the load shedding subject to the limited number of plausible outages.
2. The middle level represents the uncertainty realization in the worst possible manner within an uncertainty set, and thus it seeks to maximize the load shedding. Such uncertainty set is introduced by the following constraint [44, 45]:

$$\sum_{k=1}^{K}\left|\frac{\tilde{d}_k-\bar{d}_k}{\hat{d}_k}\right|\leq \Delta;\ \tilde{d}_k\in[\bar{d}_k-\hat{d}_k,\bar{d}_k+\hat{d}_k] \tag{6}$$

   Where $\tilde{d}_k$ is the k^{th} component of the uncertain-parameter vectors (the maximum power of generation units and bus loads), and K is the total number of buses that have uncertain power generation or load. The $\bar{d}$ is the expected value of the uncertain parameter, $\hat{d}$ is the variation from the expected value, and Δ is the "budget of uncertainty". This inequality restricts the total variation of the uncertainty realization from the expected value [46]. When $\Delta=0$, for all nodes $\tilde{d}=\bar{d}$, which means no uncertainty is considered. With increasing the budget of uncertainty, the size of the uncertainty set enlarges. This means that the resulting robust solutions are more conservative considering a larger total deviation from the expected values. Accordingly, the power system will be immunized against a higher degree of uncertainty [44]. When $\Delta=K$, the uncertainty set will be the entire hypercube defined by the intervals for each $\tilde{d}_k$. In this paper, to model the uncertainty of both loads and power generations, two independent uncertainty sets are employed.

3. Finally, the lower level models the defender as the follower and reacts against the set of out-of-service assets to mitigate the attacker's adverse consequences considering the worse-case realization of uncertain parameters from the middle-level problem. This final model will be a mixed-integer trilevel nonlinear program (MITNLP) (i.e. a trilevel max-max-max problem) which is non-convex and NP-hard.

3- Methodology

The derived multilevel non-linear model is reformulated into a single-level MILP model using the "dualize-and-combine" technique in two steps: First, the lower-level AC OPF model is approximated by an LP; then the whole bilevel model is transformed to a single-level MILP by the strong duality theorem (SDT) [47] and several proposed linearization techniques.
Herein, the phase differences between the bus voltages are assumed small enough and the voltage magnitude is close to 1 p.u. for all buses. These assumptions are practically acceptable under the normal steady-state operating condition of a power system to maintain the system far from instability [48]. Based on these assumptions, we can use the first-order approximation of Taylor's series with respect to the variables $\{V_i,V_j,c_{ij},n_{ij}\}$ for nonlinear terms of (9)-(10) (z_{ij} is the upper-level decision variable and considered as a given parameter in the lower-level problem). Appendix 2 describes the linearized model formulation.

After linearizing the lower level, the proposed mixed-integer bilevel linear problem is transformed to a single-level MI LP using the duality theory for linear programs. First, the lower-level minimization problem is replaced by its dual optimization problem. This converts the Max-Min optimization model to a Max-Max optimization model. Furthermore, similar approach will be conducted to max-max-min problem to consider uncertainty.

4- Numerical results

4-1 The IEEE RTS

The IEEE RTS consisting of 24 buses, 32 generator units, and 38 branches (lines and transformers) is used in this paper. Figure 2 shows the single-line diagram of single area IEEE RTS. Other detailed data of this test system can be found in [49]. All the simulations have been performed using a processor at 2.4 GHz with a total RAM of 32 GB. We also implement the model using the GAMS modeling language environment and the solver CPLEX [50]. In all simulations, the CPLEX relative optimality criterion was set at 0.001.

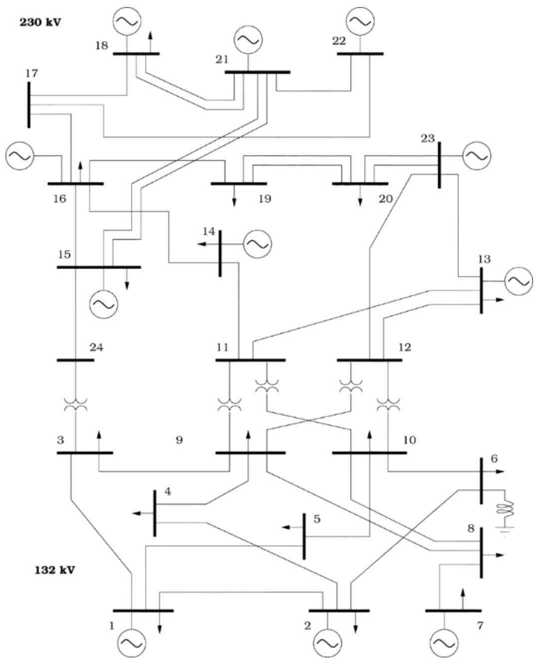

Figure 2. Single line diagram of single area IEEE 24-bus RTS.

4-2 Vulnerability analysis without load uncertainty

In this subsection, the proposed AC OPF-based MILP model is applied to the IEEE 24-bus network. Before solving the proposed MI LP model for the IEEE RTS network, the accuracy of the linearized AC OPF model in this paper and its dual optimization problem is investigated. In doing so, the exact nonlinear AC OPF is solved using MATPOWER package and the results are compared with those obtained from the proposed linearized AC OPF model. The objective function for this comparison is chosen to be the total operation cost of the generators (\$/h) in the form of $\sum_{i \in G} C_i Pg_i$, where C_i is cost coefficients of generators [51].

For the exact AC OPF and DC OPF models solved using MATPOWER package, the objective functions are 44196 \$/h and 41904 \$/h, respectively, whereas the objective function is found to be 44483 \$/h using the linearized AC OPF model. The results show an error of 5.2% as compared to the DC OPF model and a small error of 0.6% as compared to the exact AC OPF. Furthermore, the objective function is found to be 44483 \$/h using the dual optimization problem of the linearized AC OPF model (illustrating the strong-duality property of the LP model) [52].

The proposed one-level MI LP problem is applied to the IEEE RTS. Figure 3 shows the load shedding of the IEEE 24-bus system as a function of NPO (number of plausible outages). As can be seen in this figure, the optimal solutions or total load shedding is approximately the same in some cases. However, Table 1 shows small differences in some NPOs, which in turn lead to proposing different critical lines. For instance, when NPO is 3, the proposed AC OPF-based approach and the previously reported approach [34-39] find similar critical lines (i.e. 7-8). The proposed approach and the DC OPF-based approach propose different lines in the IEEE RTS model (16-19, 20-23) and (15-21, 16-17), respectively as the 2nd and 3rd critical lines. In other words, the Jaccard Similarity Index (JSI) for these two sets of lines is 0.2 and the average JSI is 0.52 for this test case.

The results shown in Table 1 can be used to compare the results of the AC OPF (columns 6 and 7) with the DC OPF (columns 2and 3). Table 1 shows “n/a” for those cases where we could not find the DC OPF-based results from the existing literature.

Using the proposed approach, the calculated load shedding is more accurate as compared to the one from the DC OPF model. Based on the simulation results for the IEEE 24-bus system, the potential critical lines are radial lines (e.g. 7-8), parallel lines (e.g. 15-21, 20-23), and the lines connecting the generation to demand zones (e.g. 11-13, 12-13). It should be noted that when all lines are out of service, the system operator is forced to shed 1607 MW which is 56% of total demand [36]. The remaining loads are directly connected to the generators' buses. The proposed MI LP model shows that this total possible system load is shed with only 13 simultaneous outages.

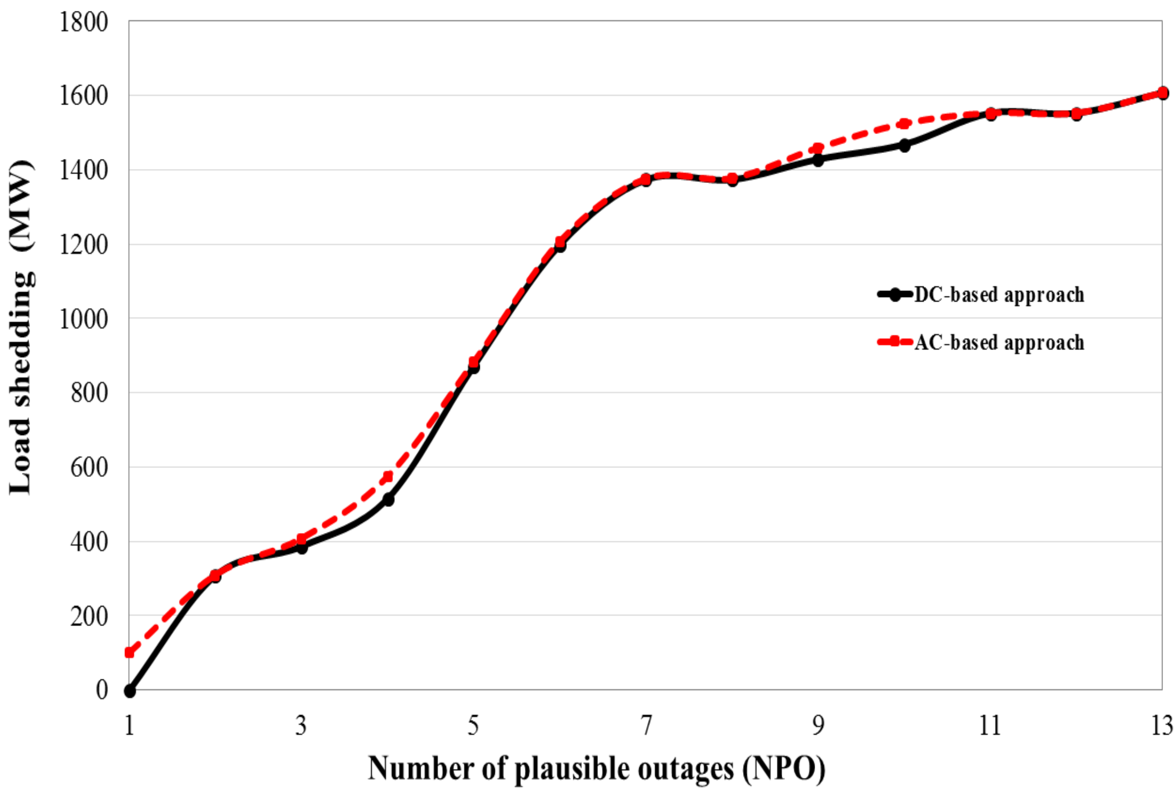

Figure 3. The optimal load shedding for IEEE 24-bus system as a function of number of plausible outages (NPO)

Table 1. The six worst-case load shedding scenarios and their related lines for the IEEE 24-bus system

NPO	DC OPF-based approach used in [34, 36]*		The DC OPF-based approach***		Proposed AC OPF-based approach		
	Critical lines	LS (MW)	Critical lines	LS (MW)	Critical lines	LS (MW)	JSI
1	n/a	n/a	-	0	7-8	99	0
2	16-19, 20-23**	309	16-19, 20-23	309	16-19, 20-23	309	1
3	n/a	n/a	7-8, 15-21, 16-17	387	7-8, 16-19, 20-23	407	0.2
4	3-24, 12-23, 13-23, 14-16	516	3-24, 12-23, 13-23, 14-16	516	7-8, 15-21, 16-17, 20-23	574	0
5	n/a	n/a	12-23, 13-23, 15-21, 16-17, 20-23	872	12-23, 13-23, 15-21, 16-17, 20-23	883	1
6	11-13, 12-13, 12-23, 15-21**, 16-17, 20-23**	1198	11-13, 12-13, 12-23, 15-21, 16-17, 20-23	1198	1-5, 11-13, 15-21, 15-24, 16-17, 20-23	1207	0.5

*This approach is also used with different objective functions and case studies in [35, 37-39].

** Two parallel circuits are considered as two independent lines in [34, 36].

*** The DC OPF-based model for cases where we do not have the benchmark results from the existing literature (see rows 1, 3 and 5).

4-3 Vulnerability analysis with load uncertainty

This subsection simulates the power system considering load uncertainty so as to assess vulnerability under load uncertainty. To consider the load uncertainty, the uncertainty budget for generation buses i.e., DR^g in the proposed model, should be set to zero in all simulations. Furthermore, the range of load variation is set to be $\hat{P}d_i = \alpha_i^d \bar{P}d_i, \forall i \in D$. In particular, in this case study α_i^d is always fixed at 0.05 (scenario I) and 0.1(scenario II) for all load buses. Moreover, the different uncertainty levels of the loads are tuned up by varying the uncertainty budget DR^d for load buses. It takes values in the range of zero (no uncertainty) to the total number of load buses (N(D) = 17). Figure 4(a) shows the LS as a function of NPO under different load uncertainties. Moreover, this figure shows that total LS increases when DR^d increases and the maximum total LS occurs when DR^d is at its maximum value 17. Furthermore, the maximum difference of LS is compared in comparison with the "no uncertainty" case when DR^d=17. Figure 4(b) shows that the maximum differences are 30% and 60% in scenarios I and II when NPO=3, respectively.

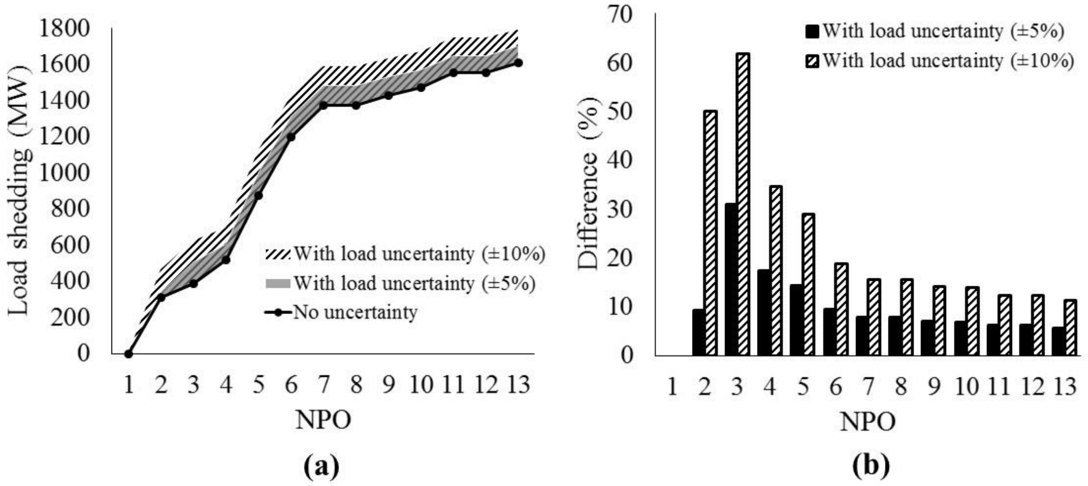

Figure 4. (a) Comparison of the optimal solutions (LS) with different levels of load uncertainty (DR^d=0 (no uncertainty) to DR^d=17 (the most conservative case)), (b) Maximum difference of LS in comparison with "no uncertainty" case when DR^d=17.

However, the load uncertainty is small, which leads to proposing different critical lines in some NPOs. For example, the model and the literature in [34, 36, 41] find similar critical lines (which are lines 16-19 and 20-23) under no uncertainty, when NPO is 2 (see Subsection 4.2). This is while this approach proposes different lines (lines 15-21 and 16-17), in a higher level of uncertainty (for example when DR^d >6 in scenario I).

5- Conclusions

This study a proposes a robust multi-level optimization problem to analyze the power system vulnerability in the context of system uncertainty. The derived multilevel non-linear model is reformulated into a single-level MILP model using the "dualize-and-combine" approach and different linearization techniques. The final MILP model has been applied to the IEEE RTS and the simulation results are carefully studied. Simulation results show that the vulnerability analysis without considering uncertainties leads to optimistic results. Moreover, increasing the level of uncertainty in the case studies leads to higher levels of LS and different critical lines in comparison with no uncertainty case.

Appendix 1: Nomenclature

Indices

i,j	Indices of buses
k	Index of regular polygon for linearizing the circle
l	Index of lines
m	Index of blocks used for piecewise linearization

Sets

D	Set of all buses with demand
G	Set of all buses with generation
L	Set of all lines
N	Set of all buses

Constants

B	Big-*M* parameter
B_{ij}^{sh}	Total line charging susceptance of line *ij* (p.u.)
C_i	Cost coefficients of generators ($/MWh)
$\text{Cos}\,\phi_i$	Power factor at bus *i*
M	Number of blocks used for piecewise linearization
n	Number of sides of a regular polygon to formulate a circle
NPO	Number of plausible outages (interdiction resources)
$Pg_i^{\max}$	Maximum of active-power magnitude for a generator at bus *i* (MW)
$Qg_i^{\max}$	Maximum of reactive-power magnitude for a generator at bus *i* (Mvar)
$Qg_i^{\min}$	Minimum of reactive-power magnitude for a generator at bus *i* (Mvar)
R(ij)	Receiving bus of line *ij*
S(ij)	Sending bus of line *ij*
$S_{ij}^{\max}$	Maximum of apparent-power magnitude for line *ij* (MVA)
Y_{ij}	Admittance of line *ij* (p.u.) $(Y_{ij} = G_{ij} + jB_{ij})$
$V_i^{\max}$	Maximum of voltage magnitude at bus *i* (V)

$V_i^{\min}$	Minimum of voltage magnitude at bus *i* (V)
$\theta_{ij}^{\max}$	Maximum of voltage-angle difference between bus i and j (Rad)
$\theta_{ij}^{\min}$	Minimum of voltage-angle difference between bus i and j (Rad)
$\Delta\theta_{ij}$	Maximum of each block width for line *ij*
Variables	
Ls_i	Active-power load shedding at bus i (MW)
Lsq_i	Reactive-power load shedding at bus *i* (MVAR)
P_{ij}	Active power-flow of line *ij* (MW)
Pg_i	Active power of generator at bus *i* (MW)
Pd_i	Active power demand at bus *i* (MW)
Q_{ij}	Reactive power-flow of line *ij* (MVAR)
Qg_i	Reactive power of generators at bus *i* (MVAR)
Qd_i	Reactive power demand at bus *i* (MVAR)
V_i	Voltage magnitude at bus *i* (V)
Z / z_{ij}	Upper-level decision variable: binary variable that is equal to 0 if line *ij* is out of service and otherwise, is equal to 1
θ_{ijm}	Width of the m$^{\text{th}}$ angle block of line *ij* (Rad)
θ_{ij}	Voltage-angle difference between bus i and j (Rad)
$\delta_{ij}^{+}, \delta_{ij}^{-}$	Positive variables used to reformulate the absolute function
T_{ij}, H_{ij}	Auxiliary variables to linearize the product of binary and continuous variables
$\lambda_{ij}, \mu_{ij}, \alpha_i, \overline{\omega}_i, \beta_i,$ $\overline{\delta}_i, \overline{\sigma}_i, \overline{\upsilon}_i, \xi_{ij}, \underline{\upsilon}_i,$ $\eta_{1,ij} \cdots \eta_{n,ij}, \chi_{ij},$ $o_{ijm}, \varphi_{ij}, \kappa_{ijm}, \varepsilon_{ij}$	Dual variables associated with their corresponding constraints

APPENDIX 2: MODEL FORMULATIONS

The AC optimal power flow (AC OPF) is proposed as the defender's tool in the lower-level problem to mitigate the attack consequences. The used formulations for AC OPF problem are:

$$Pg_{i|i\in G} + Ls_{i|i\in D} - Pd_{i|i\in D} = \sum_{j\in N} P_{ij}; \quad \forall i, j \in N : (\alpha_i) \tag{7}$$

$$Qg_{i|i\in G} + Lsq_{i|i\in D} - Qd_{i|i\in D} = \sum_{j\in N} Q_{ij}; \quad \forall i, j \in N : (\beta_i) \tag{8}$$

$$P_{ij} = z_{ij}\left(V_i^2 G_{ij} - V_i V_j (G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij})\right); \quad \forall i, j \in N : (\lambda_{ij}) \tag{9}$$

$$Q_{ij} = z_{ij}\left(-V_i^2 (B_{ij} + \frac{B_{ij}^{sh}}{2}) - V_i V_j (G_{ij}\sin\theta_{ij} - B_{ij}\cos\theta_{ij})\right); \quad \forall i, j \in N : (\mu_{ij}) \tag{10}$$

$$0 \le Pg_i \le Pg_i^{\max}; \quad \forall i \in G : (\overline{\delta}_i) \tag{11}$$
$$Qg_i^{\min} \le Qg_i \le Qg_i^{\max}; \quad \forall i \in G : (\underline{\sigma}_i, \overline{\sigma}_i) \tag{12}$$
$$\left(P_{ij}\right)^2 + \left(Q_{ij}\right)^2 \le \left(S_{ij}^{\max}\right)^2; \quad \forall i, j \in N \tag{13}$$
$$V_i^{\min} \le V_i \le V_i^{\max}; \quad \forall i \in N : (\underline{\upsilon}_i, \overline{\upsilon}_i) \tag{14}$$
$$\theta_{ij}^{\min} \le \theta_{ij} \le \theta_{ij}^{\max}; \quad \forall i, j \in N \tag{15}$$
$$0 \le Ls_i \le Pd_i; \quad \forall i \in D : (\overline{\omega}_i) \tag{16}$$
$$Lsq_i = Ls_i \tan \phi_i; \quad \forall i \in D \tag{17}$$

Where, Equations (7) and (8) are the nodal power balance equations for active and reactive powers, respectively. Equations (9) and (10) represent the line flows of active and reactive powers, respectively. Constraints (11)-(13) enforce the limits of active and reactive power generations and transmission line capacity, respectively. Voltage magnitude and voltage angle are limited using (14) and (15), respectively. Furthermore, active load shedding is limited to maximum active load at each bus in (16) and load shedding assumes constant power factor at load buses using (17). To solve our proposed MIBNLP model (1)-(17), in the next section we transform the MIBNLP to a mixed-integer linear program (MILP) which is computationally more tractable than the original MIBNLP. We then solve our proposed MILP model using the off-the-shelf Cplex solver. Dual variables associated with the constraints of the lower-level problem are shown inside parentheses.

Linearizing lower-level AC OPF model:
These first-order approximations are derived in (18) and (19) below:

$$P_{ij} = z_{ij}\left(G_{ij}\left(2V_i - 1\right) - G_{ij}\left(V_i + V_j + c_{ij} - 2\right) - B_{ij} n_{ij}\right) \tag{18}$$
$$Q_{ij} = z_{ij}\left(-(B_{ij} + \frac{B_{ij}^{sh}}{2})\left(2V_i - 1\right) + B_{ij}\left(V_i + V_j + c_{ij} - 2\right) - G_{ij} n_{ij}\right) \tag{19}$$

where, $c_{ij} = \cos\theta_{ij} \approx 1 - \frac{(\theta_i - \theta_j)^2}{2}$, and $n_{ij} = \sin\theta_{ij} \approx \theta_i - \theta_j$. Then, the quadratic function can be linearized based on the proposed method in [53] by using $2M$ piecewise linear (PWL) blocks as expressed below:

$$c_{ij} \approx 1 - \frac{(\theta_i - \theta_j)^2}{2} = 1 - \frac{\sum_{m=1}^{M}((2m-1)\Delta\theta_{ij})\theta_{ijm}}{2}; \quad m = 1 \cdots M, \forall i, j \in N : (\chi_{ij}) \tag{20}$$
$$\left|\theta_i - \theta_j\right| = \sum_{m=1}^{M}\theta_{ijm} = \delta_{ij}^{+} + \delta_{ij}^{-}; \quad m = 1 \cdots M, \forall i, j \in N : (\varphi_{ij}) \tag{21}$$
$$n_{ij} \approx \theta_i - \theta_j = \delta_{ij}^{+} - \delta_{ij}^{-}; \quad \forall i, j \in N : (\xi_{ij}) \tag{22}$$
$$0 \le \theta_{ijm} \le \Delta\theta_{ij}; \quad m = 1 \cdots M, \forall i, j \in N : (o_{ijm}) \tag{23}$$
$$\theta_{ijm} = \theta_{jim}; \quad m = 1 \cdots M, \forall i, j \in N : (\kappa_{ijm}) \tag{24}$$
$$n_{ij} = -n_{ji}; \quad \forall i, j \in N : (\varepsilon_{ij}) \tag{25}$$

where $(2m-1)\Delta\theta_{ij}$ and θ_{ijm} are the slope and the value of m^{th} block of the voltage phase difference of transmission line ij . The appropriate value for $\Delta\theta_{ij}$ can be 2π divided by 2*M*. The absolute function in (21) is linearized by introducing two positive variables δ_{ij}^{+} and δ_{ij}^{-} .

The nonlinear constraint (13) presents a circle with the radius of $S_{ij}^{\max}$. This circle is linearized by an *n*-sided convex regular polygon using following *n* equations [54]:

$$\left(\sin\left(\frac{2\pi k}{n}\right)-\sin\left(\frac{2\pi(k-1)}{n}\right)\right)P_{ij}-\left(\cos\left(\frac{2\pi k}{n}\right)-\cos\left(\frac{2\pi(k-1)}{n}\right)\right)Q_{ij}$$
$$-S_{ij}^{\max}\sin\left(\frac{2\pi}{n}\right)\le 0;\quad k=1\cdots n,\forall i,j\in N:(\eta_{k,ij}) \tag{26}$$

Linearizing the constraint (13) as suggested above, adds *n* constraints for each line. A small *n* enforces more restrictions on transmission line capacity and might lead to the problem infeasibility while a large *n* increases the number of equations and accordingly the computational burden.

Dual problem:

The Max-Max optimization model is reformulated as the following MILP maximization problem set out in (27)-(40).

$$\underset{\substack{z_{ij},\lambda_{ij},\mu_{ij},\alpha_i,\overline{\omega}_i,\\ \beta_i,\overline{\delta}_i,\overline{\sigma}_i,\overline{\upsilon}_i,\xi_{ij},\\ \underline{\upsilon}_i,\eta_{1,ij}\cdots\eta_{n,ij},\chi_{ij},\\ o_{ijm},\varphi_{ij},\kappa_{ijm},\varepsilon_{ij}}}{Max}\left(\begin{aligned}&\sum_{i,j\in N}z_{ij}\lambda_{ij}G_{ij}+\sum_{i,j\in N}z_{ij}\mu_{ij}\left(\frac{B_{ij}^{sh}}{2}-B_{ij}\right)+\sum_{i\in N}(\alpha_i+\overline{\omega}_i)Pd_{i|i\in D}+\sum_{i\in N}\beta_i Qd_{i|i\in D}\\ &+\sum_{i\in G}\overline{\delta}_i Pg_i^{\max}+\sum_{i\in G}\overline{\sigma}_i Qg_i^{\max}+\sum_{i\in G}\underline{\sigma}_i Qg_i^{\min}+\sum_{i\in N}\overline{\upsilon}_i V_i^{\max}+\sum_{i\in N}\underline{\upsilon}_i V_i^{\min}\\ &+\sum_{i,j\in N}S_{ij}^{\max}\sin\left(\frac{2\pi}{n}\right)(\eta_{1,ij}+\cdots+\eta_{n,ij})+\sum_{i,j\in N}\chi_{ij}+\sum_{i,j\in N}\sum_{m=1}^{M}o_{ijm}\Delta\theta_{ij}\end{aligned}\right) \tag{27}$$

s.t.
$$0.5\times\sum_{i,j\in N}(1-z_{ij})=NPO;\quad z_{ij}\in\{0,1\}\quad\forall i,j\in N \tag{28}$$

$$z_{ij}=z_{ji};\quad z_{ij}\in\{0,1\}\quad\forall i,j\in N \tag{29}$$

$$\lambda_{ij}-\alpha_i+\sum_{k=1}^{n}\eta_{k,ij}\left(\sin\left(\frac{2\pi k}{n}\right)-\sin\left(\frac{2\pi(k-1)}{n}\right)\right)=0:(P_{ij}) \tag{30}$$

$$\mu_{ij}-\beta_i-\sum_{k=1}^{n}\eta_{k,ij}\left(\cos\left(\frac{2\pi k}{n}\right)-\cos\left(\frac{2\pi(k-1)}{n}\right)\right)=0:(Q_{ij}) \tag{31}$$

$$-\sum_{\substack{i,j\in N\\|S(ij)=i}}z_{ij}\lambda_{ij}G_{ij}+\sum_{\substack{i,j\in N\\|R(ij)=i}}z_{ij}\lambda_{ij}G_{ij}+\sum_{\substack{i,j\in N\\|S(ij)=i}}z_{ij}\mu_{ij}(B_{ij}+B_{ij}^{sh})-\sum_{\substack{i,j\in N\\|R(ij)=i}}z_{ij}\mu_{ij}(B_{ij}+B_{ij}^{sh})+\overline{\upsilon}_i+\underline{\upsilon}_i=0:(V_i) \tag{32}$$

$$z_{ij}\lambda_{ij}G_{ij}-z_{ij}\mu_{ij}B_{ij}+\chi_{ij}=0:(c_{ij}) \tag{33}$$

$$z_{ij}\lambda_{ij}B_{ij}+z_{ij}\mu_{ij}G_{ij}+\xi_{ij}+\varepsilon_{ij}+\varepsilon_{ji}=0:(n_{ij}) \tag{34}$$

$$\alpha_i+\overline{\delta}_i\le 0:(Pg_{i|i\in G}) \tag{35}$$

$$\beta_i+\overline{\sigma}_i+\underline{\sigma}_i=0:(Qg_{i|i\in G}) \tag{36}$$

$$+\varphi_{ij}-\xi_{ij}\le 0:(\delta_{ij}^{+}) \tag{37}$$

$$\varphi_{ij} + \xi_{ij} \leq 0 : (\delta_{ij}^{-}) \tag{38}$$

$$\chi_{ij}(0.5(2m-1)\Delta\theta_{ij}) - \varphi_{ij} + o_{ijm} + \kappa_{ijm} - \kappa_{jim} \leq 0 : (\theta_{ijm}) \tag{39}$$

$$\alpha_i + \beta_i \tan\phi + \overline{\omega_i} \leq 1 : (Ls_{i|i\in D}) \tag{40}$$

In the above formulations, the primal variables associated with the different constraints of the dual optimization problem are shown in parentheses. This transformation introduces a new nonlinearity in the model, which is the product of binary and continuous dual variables ($z_{ij}\lambda_{ij}$ and $z_{ij}\mu_{ij}$) in (27), (32)-(34). This product can be linearized using two auxiliary variables T_{ij} and H_{ij} [38, 52, 55]. For instance, $T_{ij} = z_{ij}\lambda_{ij}$ can be linearized as follows:

$$\begin{aligned} &T_{ij} = \lambda_{ij} - H_{ij} \\ &-Bz_{ij} \leq T_{ij} \leq Bz_{ij} \\ &-B(1-z_{ij}) \leq H_{ij} \leq B(1-z_{ij}) \end{aligned} \tag{41}$$

Where, B is a suitable large constant. The final proposed MILP model can be solved using the high performance, efficient, and reliable off-the-shelf solvers such as Cplex [56, 57]. These solvers can efficiently solve our MILP model to the desired level of accuracy. They can also provide the certificate of optimality of the solution. This is while the original MIBNLP model is an NP-hard optimization problem with no guarantee of finding the global solution [48].

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