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Phase Angles as a Proxy for Voltage Stability

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SUMMARY

A previous paper [1] discussed the calculation of a Triangle Voltage Stability Index (TVSI) for monitored alternating-current circuits using voltage data from a PSSE load flow study. The analysis provides TVSI values for monitored transmission circuits in the Bulk Electric System under varying power transfers and contingencies. TVSI provides an indication of the closeness of the load voltage to potential voltage collapse.

To provide situational awareness to system operators, AEP proposes monitoring the phase angle across a low loss EHV overhead circuit operating in a system environment and comparing the angle to an established phase angle loci as a proxy for TVSI. This monitoring could be independent of the line loading or the associated line impedance.

To establish the phase angle loci we use a Distributed Parameter Transmission Line [2] as an Electrical Free Body Diagram (EFBD). We describe a new innovative voltage stability triangle that encapsulates the transmission line propagation constant ($\boldsymbol{y}$) and characteristic impedance (Z_c) of the Distributed Parameter Transmission Line. The triangle allows us to assign a TVSI value and determine the associated phase angle loci for transmission lines with 80 -250km and base voltages of 345kV -765kV. These line lengths account for the majority of circuits in AEP.

Finally, this paper compares the phase angles in a PSSE TVSI study to the established phase angle loci.

KEYWORDS

Phase angle, phase angle loci, triangle, surge impedance, propagation constant

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INTRODUCTION

The power flow on an AC transmission line can be impacted by thermal, voltage magnitude, voltage-drop, and steady-state stability limitations. This paper deals with the steady-state voltage limitations [3] and using phase angles as a proxy. Existing real time tools in the control room address the thermal, voltage magnitude, and voltage drop limitations. Reference [4] notes voltage stability is related to phase angles. In addition, the power component of a traditional PV curve is associated with phase angles across a circuit.

A ***bold italic variable*** denotes a complex phasor quantity; otherwise, the variable represents the magnitude of the phasor quantity.

DISTRIBUTED PARAMETER TRANSMISSION LINE

Equation (1) from the literature [2] describes the voltage for a Distributed Parameter Transmission line. Figure 1 portrays the line serving a complex load $\boldsymbol{S}$ with load voltage $\boldsymbol{V_{lg}}$.

$$\textbf{\textit{(1)}}\ \boldsymbol{E_{lg}} = \boldsymbol{V_{lg}} * \boldsymbol{cosh}(\boldsymbol{Y} *\text{km}) + \boldsymbol{Z_c} * \boldsymbol{I} * \boldsymbol{sinh}(\boldsymbol{Y} *\text{km})$$

Where
$\boldsymbol{E_{lg}}$ – Line to ground source voltage
$\boldsymbol{V_{lg}}$ – Line to ground load voltage
$\boldsymbol{Y}$ – Propagation constant per km
$\boldsymbol{Z_c}$ – Characteristic Impedance in ohms
$\boldsymbol{I}$ – Load/Line Current in Amperes
km – Distance from load end towards source end in km

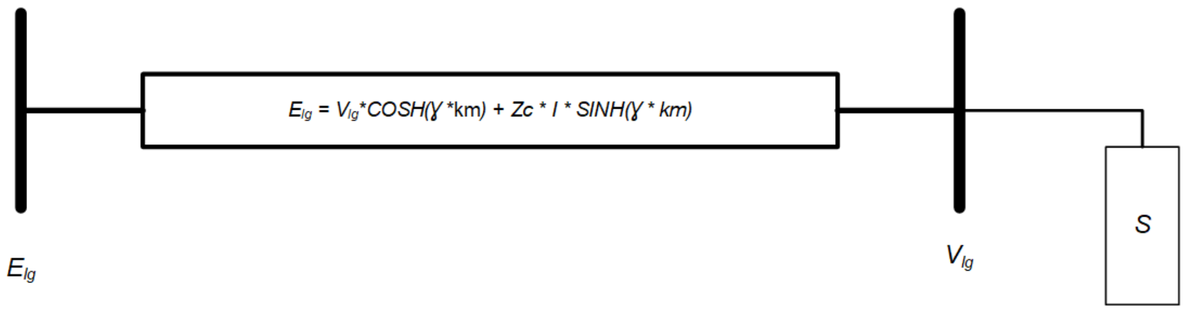


Figure 1 – Transmission Line serving a Load S with Voltage V

In subsequent deliberations, this paper uses a per unit (PU) system to express voltages and impedance.

VOLTAGE STABILITY TRIANGLE

The system in Figure 1 has two per unit (PU) load voltage solutions $\boldsymbol{V_u}$ and $\boldsymbol{V_l}$. In the context of a PV curve, $\boldsymbol{V_u}$ is the upper voltage solution while $\boldsymbol{V_l}$ is the lower voltage solution. TVSI equals the ratio of V_l / V_u or equivalently $|\boldsymbol{E} - \boldsymbol{V_u}|$ / Vu. [1,5,6,7] We have reached a voltage bifurcation point when TVSI equals one.

Figure 2 displays a new novel triangle for the Distributed Parameter Transmission Line. The triangle allows for the calculation of the load voltage phase angle Φ_u given the source voltage E, the W factor described below, the load voltage V_u, and the TSVI magnitudes.

The E, V_u, and V_l voltages displayed on the triangle exterior sides contain the PU voltage magnitudes. The angles Φ_u, Φ_l, and $\pi - \beta$ at the triangle vertices represent the voltage angles for V_u and V_l and an angle related to the effective load / line parameter factor which captures the load served through the distributed line impedance.

We summarize the parameters below.
E – PU Source Voltage magnitude
V_u – PU Load Voltage magnitude (physically realizable)
Φ_u – Phase angle of $\boldsymbol{V_u}$ relative to E at zero degrees
V_l – PU Load Voltage magnitude (not physically realizable)
Φ_l – Phase angle of $\boldsymbol{V_l}$ relative to E at zero degrees
W – |***cosh***(***Y****m)| magnitude
    Effectively stretches the source voltage E magnitude since the W magnitude is less than one for ***Y*** parameters noted in this paper.
m – Length of line in km

Equation (2) defines the parameter $\boldsymbol{\rho @ \beta}$. $\boldsymbol{\rho @ \beta}$ is a factor that accounts for the load served via the distributed line impedance.

$$(2)\ \boldsymbol{\rho @ \beta = S^{*} * Z/W}$$

Where

S is the complex load ***(P+jQ)*** in PU

Z equals Z_c ****sinh***(***Y*** *m)/(kV^2/100) is in PU

W equals ***cosh***(***Y****m)

In addition, $\boldsymbol{\rho @ \beta}$ equals the product of the complex conjugate load voltages (3).

$$(3)\ \boldsymbol{\rho @ \beta = V_u^{*} * V_l^{*}}$$

Although not demonstrated in this paper, the interior of the triangle allows for the direct calculation of the load voltages $\boldsymbol{V_u}$ and $\boldsymbol{V_l}$. given ***E***, ***W***, Z_c, and ***S***. This paper deals with line lengths of 80 -250km. For long lines, the W parameter magnitude can approach 0.5. This effectively stretches the source voltage E by a factor of 2. At this point the phase angle of $\boldsymbol{V_u}$ will be small. In the AEP system, the magnitude of W is bounded to 0.95 to 1 and this is not an issue.

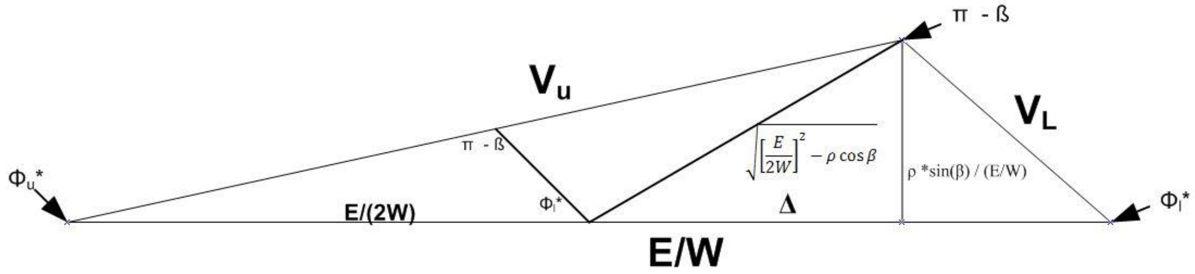


Figure 2 – Distributed Parameter Transmission Line Voltage Stability Triangle

Application of the Triangle

Table 1 list the transmission line propagation constant $\boldsymbol{Y}$ and characteristic impedance Z_c for 345, 500, and 765kV EHV lines. The $\boldsymbol{Y}$ and Z_c parameters are from [3]. The table also lists the $\boldsymbol{W}$ factor for 80 and 250km lines. $\boldsymbol{W}$ magnitude effectively stretches the source voltage E in Figure 2. $\boldsymbol{W}$ also affects the phase angles of the load voltages.

Nominal Voltage	345kV	500kV	765kV
$\boldsymbol{Y}$ / km * 10^{-3}	0.066 +j1.29	0.057 +j1.3	0.025 +j1.28
Z_c (ohms)	285	250	257
$\boldsymbol{W}$ for 80km	0.9947@0.0313°	0.9946@0.0273°	0.9948@0.0118°
$\boldsymbol{W}$ for 250km	0.9486@0.3159°	0.9478@0.2751°	0.9493@0.1187°

Table 1 – Distributed Transmission Line Parameters

This analysis demonstrates that for a given TVSI, the phase angle of the load voltage solution $\boldsymbol{V_{un}}$ has a narrow range for line lengths 80km to 250km and for a prescribed source and load voltage. A larger load $\boldsymbol{S}$ accompanies a larger TVSI but at the cost of system security.

Given:

- 345kV distributed line parameters $\boldsymbol{Y}$ and Z_c from Table 1
- E - 1PU
- V_u - 0.95PU
- TVSI (V_l/V_u) - 0.5 to 0.9
- Line lengths – 80km and 250km

Using the triangle in Figure 2, we calculate the load voltage $\boldsymbol{V_u}$ and $\boldsymbol{V_l}$ phase angles and the β angle. The factor W effectively elongates the source voltage E by about 5% for the 250km line.

The $\boldsymbol{W}$ factor also introduces a phase angle shift in the load voltages. Consequently, the load voltage solutions to the system are given by $\boldsymbol{V_{un}}$ and $\boldsymbol{V_{ln}}$. (4) Substitution of the load voltages $\boldsymbol{V_{un}}$ and $\boldsymbol{V_{ln}}$ into a PU version of (1) results in the 1 PU E source voltage.

(4)

$$\boldsymbol{V_{un}} = \boldsymbol{V_u} * W/\boldsymbol{W}$$

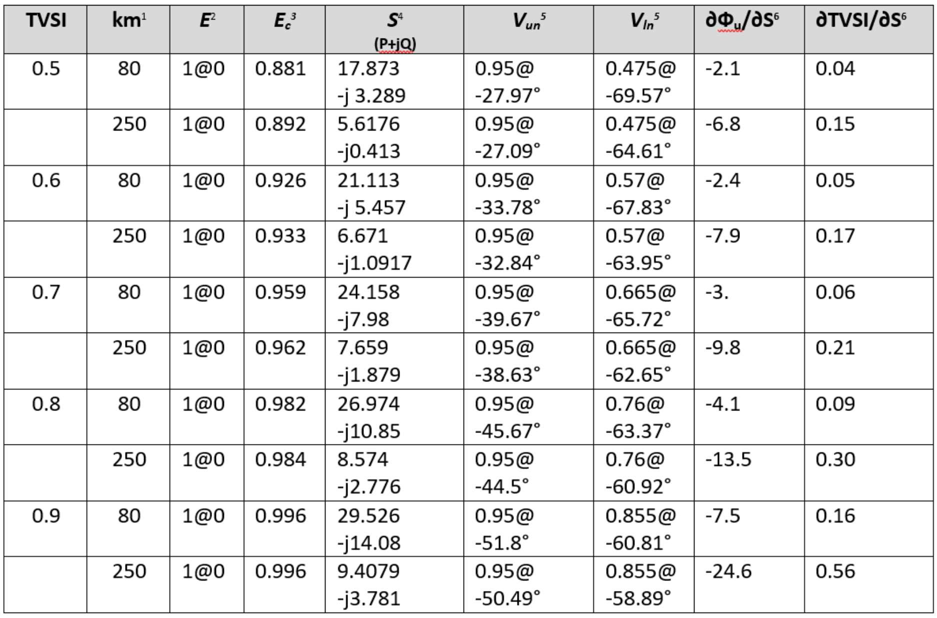
$$\boldsymbol{V_{ln}} = \boldsymbol{V_l} * W/\boldsymbol{W}$$

Table 2 summarizes the study results for the 80km and 250km 345kV lines. Note for a given TVSI, the $\boldsymbol{V_{un}}$ phase angles are limited to a narrow region. As an example, the $\boldsymbol{V_u}$ phase angle associated with TVSI 0.6 ranges from -32.8 to -33.8 degrees.

Other factors associated with a larger TVSI include the following:

1. Increases the critical E_c source voltage necessary to support the load $\boldsymbol{S}$.
    - If E is less than E_c then the system does not have a solution.
2. Increases the partial derivative ∂TVSI/∂S and $\partial\Phi_u/\partial S$ magnitudes.

Items 1 and 2 suggest TVSI larger than 0.7 may be harder to manage on a system. Consequently, subsequent analysis in this paper only reports on TVSI less than or equal to 0.7.



TVSI	km[1]	E[2]	E_c[3]	S[4] (P+jQ)	V_{un}[5]	V_{ln}[5]	$\partial\Phi_u/\partial S$[6]	$\partial TVSI/\partial S$[6]
0.5	80	1@0	0.881	17.873 -j 3.289	0.95@ -27.97°	0.475@ -69.57°	-2.1	0.04
	250	1@0	0.892	5.6176 -j0.413	0.95@ -27.09°	0.475@ -64.61°	-6.8	0.15
0.6	80	1@0	0.926	21.113 -j 5.457	0.95@ -33.78°	0.57@ -67.83°	-2.4	0.05
	250	1@0	0.933	6.671 -j1.0917	0.95@ -32.84°	0.57@ -63.95°	-7.9	0.17
0.7	80	1@0	0.959	24.158 -j7.98	0.95@ -39.67°	0.665@ -65.72°	-3.	0.06
	250	1@0	0.962	7.659 -j1.879	0.95@ -38.63°	0.665@ -62.65°	-9.8	0.21
0.8	80	1@0	0.982	26.974 -j10.85	0.95@ -45.67°	0.76@ -63.37°	-4.1	0.09
	250	1@0	0.984	8.574 -j2.776	0.95@ -44.5°	0.76@ -60.92°	-13.5	0.30
0.9	80	1@0	0.996	29.526 -j14.08	0.95@ -51.8°	0.855@ -60.81°	-7.5	0.16
	250	1@0	0.996	9.4079 -j3.781	0.95@ -50.49°	0.855@ -58.89°	-24.6	0.56

Table 2 – Load Voltage Phase Angles for Varying TVSI and 345kV Line Length

Notes:

1. 80km line would likely be thermally limited
2. Calculate $\boldsymbol{E}$ using a PU version of (1) from $\boldsymbol{V_{un}}$ or $\boldsymbol{V_{ln}}$.
3. $\mathbf{E_c}$ is the critical source voltage magnitude. Source voltages less than E_c are not capable of supplying the load $\boldsymbol{S}$.
4. $\boldsymbol{S}$ is calculated from $\boldsymbol{V_u}$ and $\boldsymbol{V_l}$ using (2) and (3).
5. $\boldsymbol{V_{un}}$ and $\boldsymbol{V_{ln}}$ are the load voltage solutions calculated from (4). This account for the fact the line length rotates the load voltage by a fraction of a degree.
6. Derivative calculated numerically at operating point $\boldsymbol{S}$, $\boldsymbol{V_{un}}$, and $\boldsymbol{V_{ln}}$ for the given TVSI and line length. Larger TVSI have an increasingly non-linear response to the change in phase angle / TVSI for a change in the load S.

Comparing the Voltage Phase Angle Loci for 345kV – 765kV Lines

This section demonstrates for a prescribed TVSI level and line length the V_{un} phase angles are nearly independent of the base voltage level. Table 3 displays the study results for 765kV, 500kV, and 345kV lines for TVSI 0.5, 0.6, and 0.7. The E and V_u voltages are at 1 and 0.95PU respectively.

TVSI=>	******** 0.5 ********			******** 0.6 ********			******** 0.7 ********		
kV=>	345	500	765	345	500	765	345	500	765
Km	Φ_u	Φ_u	Φ_u	Φ_u	Φ_u	Φ_u	Φ_u	Φ_u	Φ_u
80	-27.97°	-27.96°	-27.95°	-33.78°	-33.77°	-33.76°	-39.67°	-39.66°	-39.65°
250	-27.09°	-27.03°	-26.91°	-32.84°	-32.77°	-32.66°	-38.63°	-38.56°	-38.45°

Table 3 – Load Voltage Phase Angles for Varying TVSI, Line kV, and Line Length

LOAD VOLTAGE PHASE ANGLE LOCI

Inspection of the triangle in Figure 2 indicates the voltage V_u phase angle is a function of E,W,V_u, and V_l magnitudes. Figure 3 contains a plot of the phase angles vs TVSI for a given source and load voltage range, and two circuit lengths over a TVSI range. The plots use these parameters.

E – 0.95 -1.05PU (Normal voltage range)

V_u – 0.95 -1.05PU (Normal voltage range)

W - Table 1 345kV 80km and 250km

TVSI – 0.3 -0.7

In general, E greater than V_u results in smaller phase angle magnitudes as reflected in the top two curves. Conversely, E less than Vu results in larger phase angle magnitudes as indicated in the two bottom curves. For a given TVSI, the phase angle loci is bounded by the top and bottom curve(s). As an example, for a TVSI of 0.6 the phase angle loci ranges from -31.3 to -36.3 degrees.

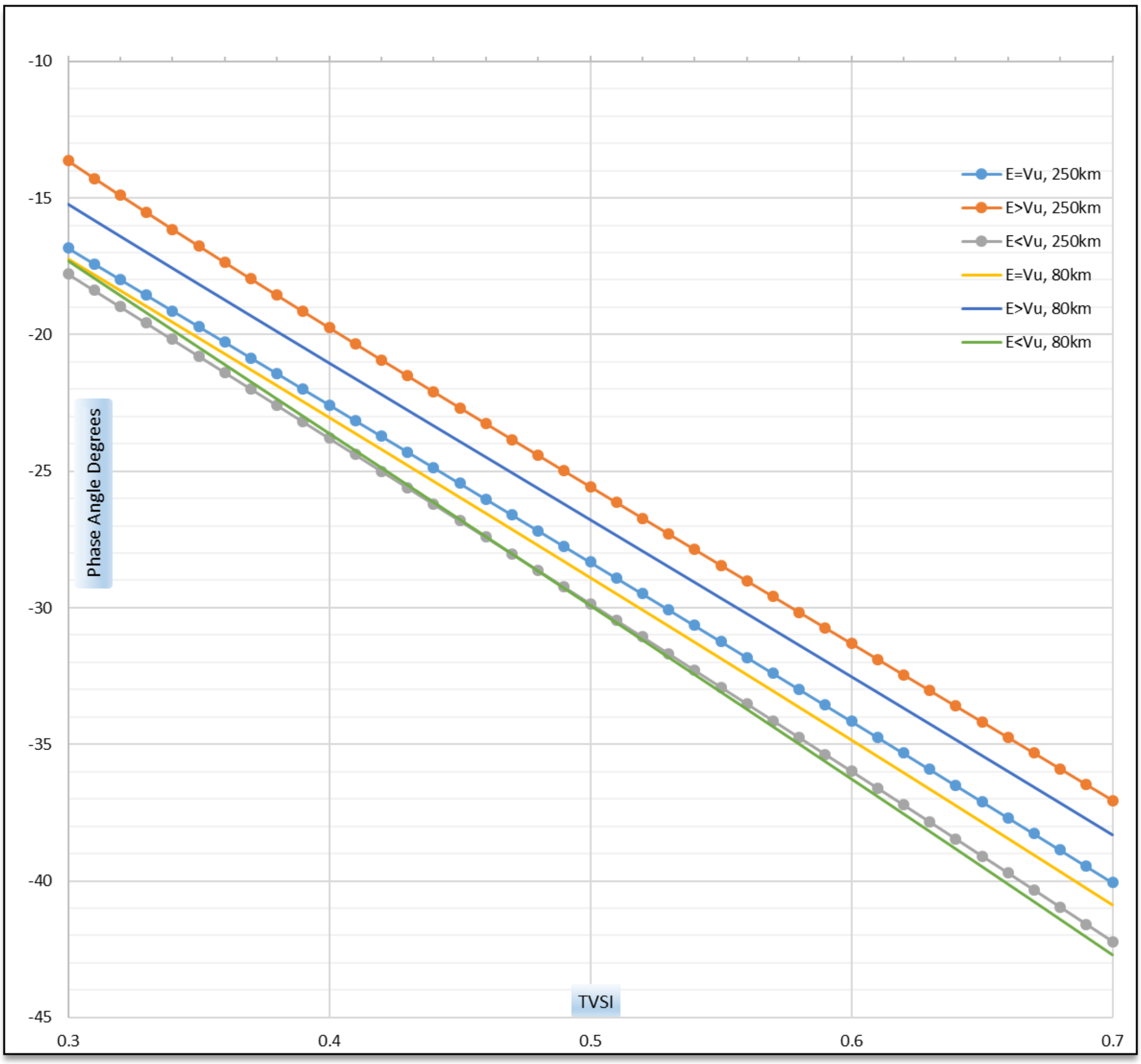


Figure 3 – Phase Angle vs TVSI

Comparison of the PSSE Phase Angles and the Triangle Phase Angle Loci

Table 5 shows the comparison of the PSSE Phase Angles vs the Triangle Phase Angle Loci for a given PSSE TVSI. Each TVSI value is calculated from the PSSE case and contingency for the From bus – To bus circuit. This list represents a diverse group of circuit elements from the AEP East and West systems. The comparison demonstrates that there is excellent agreement between the PSSE phase angles and the Triangle Phase Angle Loci for a given PSSE TVSI.

PSSE TVSI Study							**Phase Angle Loci for given PSSE TVSI**
From bus	**To bus**	**TVSI**	**BASE kV**	**Difference Angle in Degrees**	**Contingency**	**Case**	**Angle Range Degrees[2]**
A	C	0.6784	138	-38.9°	N –V 345kV + L –V 345kV	2018 Summer SPA South –North transfer	-35.8°, -40.6°
N	V	0.45817	345	-26.3°	L –V 345kV + M –F 345kV	2018 Summer SPA South –North transfer	-23.2°, -27.3°
P	V	0.41695	345	-23.6°	H –S 345kV + E	2018 Summer SPA South –North transfer	-20.7°, -24.7°
N	W	0.34818	345	-19.2°	L –W 345kV + R –C 345kV	2018 Summer SPA South –North transfer	-16.6°, -20.3°
D	S	0.65237	345	-37.9°	J –R 765kV + D –E 345kV	2018 Summer SPA West –East transfer	-34.3°, -39.6°
D	S	0.53417	345	-30.7°	J –R 765kV	2018 Summer SPA West –East transfer	-27.5°, -32.1°
K	M	0.4547	345	-26.4°	C –J 765kV	2018 Winter SPA North –Southeast transfer	-23.°, -27.1°
K[1]	M	0.3805	345	-21.9°	C –J 765kV	2018 Winter SPA North - Southeast transfer	-18.6°, -22.4°

Table 5 - Comparison of PSSE case TVSI Study to the Triangle Calculated Phase Angles

Notes:

1. Line with 30% series compensation
2. Used Figure 3 data to determine Phase Angle Loci for a given TVSI.

CONCLUSIONS:

To provide situational awareness to system operators, AEP proposes using phase angles as a proxy for voltage stability using EHV voltages in the normal range of 0.95 -1.05 PU. Existing real time tools in the control room address voltage magnitude and voltage drop limitations outside the normal voltage range.

There is close agreement between the PSSE phase angles and the Triangle Phase Angle Loci for a given PSSE TVSI. The paper recommends limiting TVSI levels to 0.7 or smaller since higher TVSI levels may be harder to manage. Figure 3 contain plots of the phase angles vs TVSI for the normal voltage range.

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