



21, rue d'Artois, F-75008 PARIS  
<http://www.cigre.org>

## CIGRE US National Committee 2019 Grid of the Future Symposium

### **Volt/Var Optimization by Smart Inverters and Capacitor Banks**

**G. ALVAREZ, C. C. LIU**  
Virginia Tech  
USA

**J. PETTI, R. LIU**  
Dominion Energy  
USA

#### **SUMMARY**

The high penetration of power electronic based distributed energy resources (DERs) has increased the importance and attention given to voltage security of distribution systems. Voltage control in the electrical power system is critical for a proper operating condition. Therefore, distribution systems must have the ability to maintain a secure voltage profile. Using inverters for Volt/VAR control (VVC) can provide a faster response for voltage regulation than traditional voltage regulation devices, such as transformer load tap changers and voltage regulators. The primary objective of this paper is to demonstrate how smart inverters can be used to eliminate the voltage deviation by solving a mixed-integer quadratic program to determine the amount of reactive power that should be injected or absorbed at the appropriate nodes. The proposed method incorporates capacitor banks connected to the network and determines whether to turn on or off the capacitor bank for voltage regulation. These processes will be demonstrated in several cases that are focused on mitigating voltage-dips and swells.

#### **KEYWORDS**

Big-M Method, Distributed Energy Resources, Mixed-Integer Quadratic Programming, Photovoltaic Array, Volt/Var Optimization, Smart Inverters

## I. Introduction

The increase of power electronic based DER interconnections on the distribution system has shown the ability to impact voltage regulation and performance. Traditionally, VVC is conducted by transformer load tap changers, voltage regulators, and capacitor banks. The increase of power electronics in the electric power system (EPS) has increased the level of interest to other VVC methods, e.g., STATCOM and the inverters that interface photovoltaics (PVs) and battery energy storage system (BESS) to the grid. As power electronic technology advances, smart solar inverters now have the capability to stay connected to the grid during small disturbances rather than disconnecting from the grid. As per the new IEEE 1547 standard, distributed generation that is inverter-based is required to stay connected to the grid during voltage and frequency ride through [1]. Inverter controls have been designed to inject/absorb reactive power based on the deviations in voltage magnitude.

Power system operators maintain power system security, reduce power losses and the generation cost by utilizing the optimal power flow (OPF) analysis to operate the transmission system [2]. Power flow analysis and modeling is analyzed in [3]-[6]. The branch flow model with two relaxation steps analyzes the optimal power flow (OPF) for radial and meshed networks [3]. Since, most distribution circuits are radial networks, reference [4] proposes a robust method to solve the distribution system power flow for a radial feeder. The phase angles of the currents and voltages are omitted. A nonlinear model generally has a high computational cost. If the power losses are ignored in the distributed flow model, a linearized distribution flow model (*LinDistFlow*) is obtained [5]. A linearized model leads to a convex optimization problem. However, a linearized model incorporating capacitor banks has not been considered.

Reference [6] proposes a technique for volt/var optimization incorporating the interaction between inverters and capacitors using conditional-value-at-risk to obtain a convex approximation. In reference [7], an algorithm is designed to minimize losses and flatten the voltage profile. Reference [8] obtains the reactive injection values of the inverters using nonlinear control policies determined by kernel-based learning. In this case, the communication overhead is reduced to a 30-min basis. To coordinate existing VVC devices and inverters, reference [9] proposed a hybrid VVC scheme.

This paper proposes a method to determine the optimal reactive power injection or absorption needed from either the inverter or capacitor bank to minimize voltage deviations using mixed integer quadratic programming. Gurobi/MATLAB is used to implement the algorithm that optimizes the reactive power injection or absorption when there is a load pick up during restoration or load shedding when there is an insufficient power supply. A case study based on a 13-node distribution system is presented with DERs connected to multiple buses and two capacitor banks connected to the node. This algorithm is implemented in an environment of centralized dispatch. This paper proposes a mixed-integer quadratic programming problem that is solved using the Big-*M* method and obtains the global solution.

The rest of the paper is organized as follows. Section II formulates the problem that is being addressed. Section III describes the optimization method used to determine the reactive power injection. Section IV describes the 13-node feeder model and the two simulation cases that are conducted. Section V provides the conclusion and future extension of this work.

## II. Problem Formulation

The two objectives of this research are to minimize the voltage deviations from the nominal value by controlling reactive power injections from inverters and, if there is a capacitor bank connected to a node, to determine whether to turn on or off the capacitor bank. The proposed Volt/VAR optimization method is based on the approximate linearized distribution flow (LDF) model (1a) proposed in [10]. This method is adopted for this project due to its high computational performance. The radial single-phase distribution system has  $N+1$  node.  $N_0$  corresponds to the substation node, as seen in Figure 1. The voltage at the substation node is the nominal voltage.  $v_0 1_N$  denotes the nominal squared voltages. Variable  $v_n$  represents the squared voltage magnitude of the nodes  $n$ , excluding the substation node.

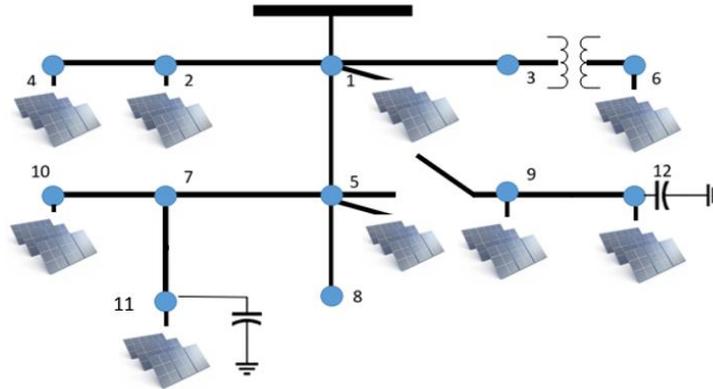


Figure 1 IEEE 13 Node Distribution System

DERs are placed at every nonzero node in the network according to the IEEE 13 node test feeder. The active and reactive power are generated by the DERs connected to the grid at node  $n = \{1, \dots, N\}$ , represented by  $(p_g^n, q_g^n)_{n=1}^N$ .  $(p_c^n, q_c^n)_{n=1}^N$  is the power consumed by the load at node  $n$ . It is assumed that  $p_g, p_c$  and  $q_c$  are known quantities.

The active and reactive power injections are:

$$\begin{aligned} p &= p_g - p_c \\ q &= q_g - q_c \end{aligned}$$

Matrix  $A$  is the reduced branch-bus incidence matrix of the distribution system. The resistance and reactance of the lines in vector  $r$  and  $x$  are given based on the IEEE 13 node test feeder configuration impedance.  $z = r+jx$  and  $Z = \text{diag}(z)$ , when equation (1a) is multiplied by  $A^{-1}$  and then  $S = A^{-T}S$  is substituted. Using the property  $A^{-1} a_0 = -1_N$  will simplify the equation to (1b).  $R$  and  $X$  is a symmetric positive definite matrix and is expressed in (1c-1d); see [10] for a similar linear approximation model.

$$Av = 2\text{Re}[Z^*S] - a_0v_0 \quad (1a)$$

$$v_n \simeq Rp + Xq + v_01_N \quad (1b)$$

$$X = 2A^{-1}\text{diag}(x)A^{-T} \quad (1c)$$

$$R = 2A^{-1}\text{diag}(r)A^{-T} \quad (1d)$$

When the active and reactive power injections,  $p$  and  $q$ , are given, the voltage deviation can be found by a linear equation as per [5]

$$\Delta v \simeq Rp + Xq \quad (1d)$$

$$\Delta v = \begin{bmatrix} v_1 - v_0 \\ \vdots \\ v_N - v_0 \end{bmatrix}$$

$Rp + Xq$  demonstrates the voltage drop in the line due to the line impedance. Where  $\Delta v$  represents the deviation in squared voltages. When the reactive power injection of the capacitor bank is incorporated, the new linearized distribution flow model is given by:

$$\Delta v \simeq R(p_g - p_c) + X((q_g - q_c) + q_{cb}) \quad (2a)$$

The following values are given as measurements:

$p_g$ : Active solar generation

$p_c$ : Active load consumed

$q_c$ : Reactive load consumed

There are two optimization variables:

$q_{cb}$ : Reactive power produced by the capacitor banks  
 $q_g$ : Reactive solar generation

The objective is to minimize the voltage deviations by minimizing the  $l_2$ -norm of vector  $\Delta v$ .

$$\min \|\Delta v\|_2^2 \quad (2b)$$

Based on the apparent power capacity of the inverters and the constraint not to create reverse power flow, the following constraints are enforced. The limits are given:

$$p_g^2 + q_g^2 \leq (S_g^{max})^2 \quad (3)$$

IEEE 1547 standard [1] states that voltage distortion limits for a node voltage at the point of common coupling (PCC) should be 3% for any voltage that is 69 kV or below.  $dv_n$  is the absolute deviation voltage. Hence,

$$dv_n = \sqrt{\Delta v + 1} - 1 \quad (4a)$$

$$dv_n^{min} \leq dv_n \leq dv_n^{max} \quad \forall n \quad (4b)$$

$$-0.03 \leq dv_n \leq 0.03 \quad (4c)$$

Given the constraints and objective function, the computational problem can be solved by mixed integer quadratic programming.

There are switched capacitor banks within the network. When the capacitor bank is switched on it injects reactive power to the bus and when it is off it does not. Let  $x_s$  be a binary variable that is used to indicate if the capacitor bank is switched on or off. Equation (5) expresses what the value of  $q_{cb}$  will be depending on whether  $x_s$  is 0 or 1.  $\omega$  is the angular velocity at 60Hz and  $v_{cb}$  is the voltage of the node connected to the capacitor bank.

$$q_{cb} = \begin{cases} v_{cb}\omega C, & x_s = 1 \\ 0, & x_s = 0 \end{cases} \quad (5)$$

$v_{cb}$  is an unknown variable and  $x_s$  is an unknown binary variable. The value of  $q_{cb}$  is given by

$$q_{cb} = v_{cb}\omega C x_s \quad (6a)$$

Note that  $q_{cb}$  is a nonlinear equation. A technique used to linearize equation (6a) is the Big- $M$  method which introduces different inequalities and the large positive number  $M$ . Consider the constraint  $q_{cb} = v_{cb}\omega C x_s$ , if  $v_{cb}\omega C$  is bounded with  $v_{cb}\omega C \in [-M, M]$ , the new constraints can also be expressed as

$$-Mx_s \leq q_{cb} \leq x_s M \quad (6b)$$

$$-M(1 - x_s) \leq q_{cb} - v_{cb}\omega C \leq (1 - x_s)M \quad (6c)$$

$$x_s \in \{0,1\} \quad (6d)$$

If  $x_s = 1$ , then constraint (6c) becomes an equality and  $q_{cb}$  is less than the upper bound  $M$ , due to constraint (6b). When  $x_s = 0$  then the first inequality (6b) ensures  $q_{cb} = 0$ . Minimizing the voltage deviations can be solved using the gurobi solver in YALMPI and interfaces with MATLAB by conducting mixed integer quadratic programming computation.

### III. Numerical Tests

According to the IEEE 13-node test feeder, nodes 3, 7, and 8 do not have any load connected to it as shown in Figure 1. In this system, the capacitor bank injection at node 11 is 100 kVAR and 200 kVAR at node 12, respectively. In this distribution model, aluminium conductors steel-reinforced (ACSR) of the aluminium area 556,500 26/7 cmil is used. The real data used in this case study was taken from the Pecan St dataset [12]. The minute-sampled PV generation and loads data is taken on May 1, 2013, at 8:00 am. The load was scaled so that the monthly peak will be equivalent to 30% of the benchmark load values. A random lagging power factor between 0.85 to 0.9 is used to generate the reactive loads.

### IV. Simulation Cases

Since diverse loads have been added to the EPS and load forecast is difficult, it is desirable to have a continuous device that regulates the voltage profile. The penetration of inverter based DERs is increased in the distribution system and, therefore, using the inverters for voltage regulation is a feasible solution. Switched capacitor banks are already integrated within the distribution system and so enabling these two devices to work together is the objective of this study. In this simulation case, the load demand of the customer is increased, and it is intended to evaluate how the inverter and capacitor bank in the system can help eliminate the voltage deviations. The results are analysed while there is a decrease in load to observe how the inverter and capacitor bank will react and whether the voltages will stay within its constraints. If either one of the two scenarios occur, there can be an imbalance in power generation, leading to under-frequency, angle instability, voltage violations, and overloading other generation units in the power system.

#### IV.A. Simulation Case 1

Both simulation cases use a 13-node radial test feeder with DERs connected to 9 out of the 12 nodes. It is not practical to expect a DER at every node of the distribution system. Other techniques such as LTC or a capacitor bank are used to regulate the voltage. This simulation case is developed to observe if the capacitor bank should be switched on or off, depending on the

amount of reactive power that is needed for voltage regulation. The initial voltage and reactive power are shown in Figure (2a-2b).

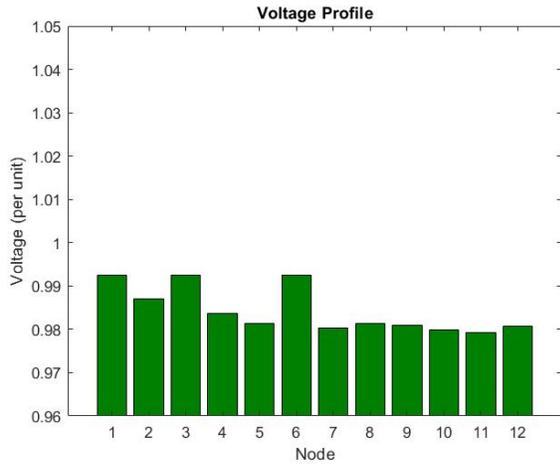


Figure 2a Voltage at each node

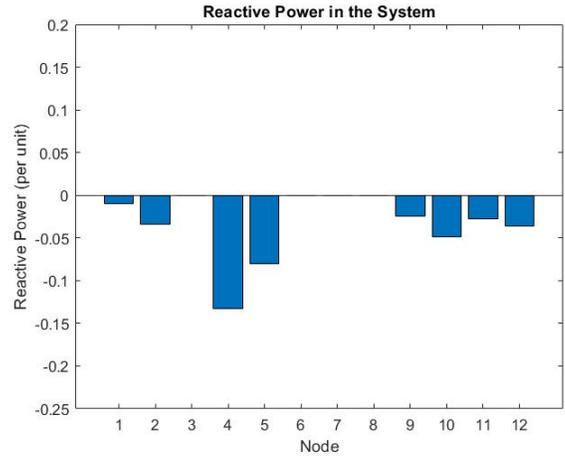


Figure 2b Reactive power at each node

Case 1 represents what may occur when there is a high customer demand. As the power consumption increases, low voltage violations occur as illustrated by Figure (3a). The power consumed in the entire system is increased by 50%, causing a voltage dip. When using the controller, the voltage remains within tolerance as shown in Figure (3b).

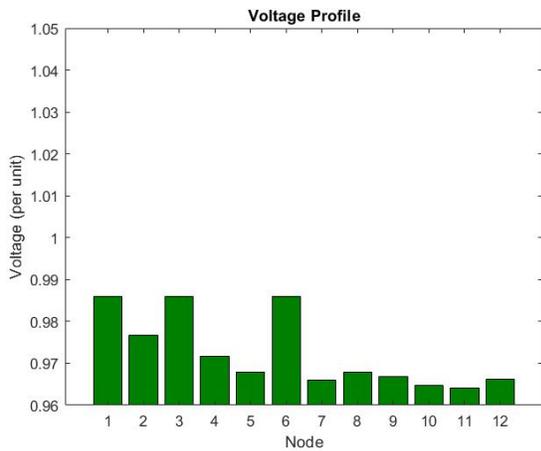


Figure 3a Voltage of profile without controller implemented during load increase

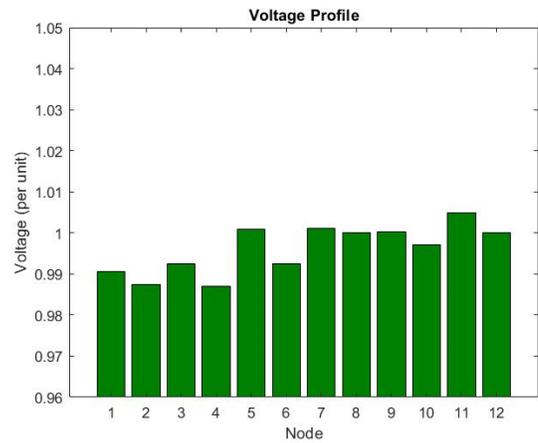


Figure 3b Voltage of profile with controller implemented during load increase

Figure (4a-4b) demonstrates the output reactive power of the DER inverters and capacitor bank at its corresponding node. The active power generated is kept constant according to the measured value at that time. Figure 4a occurs when no controller is implemented. Figure 4b includes control, when the power consumption increases the amount of reactive power absorption decreases to mitigate the voltage deviations. The capacitor bank at node 11 is switched on because the closest inverter has reached the maximum level of reactive power output capacity.

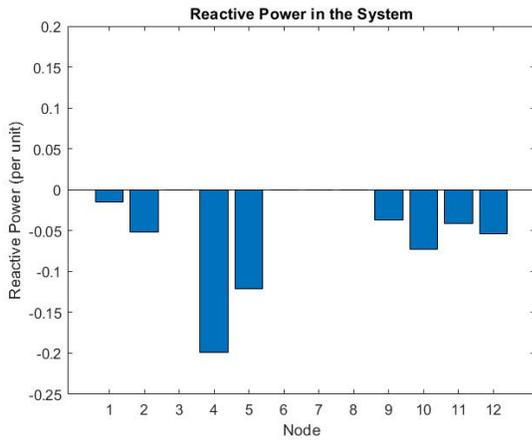


Figure 4a Reactive power at each node during load increasing without a controller

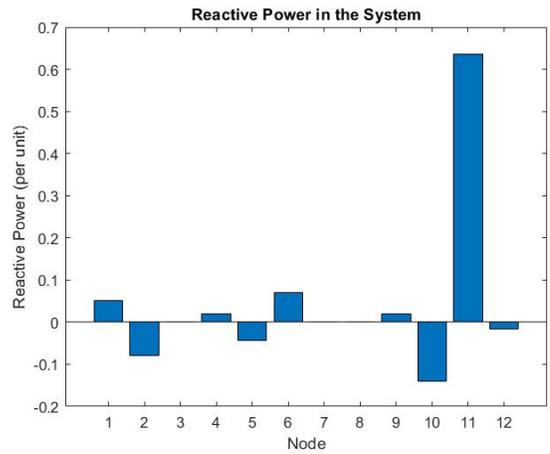


Figure 4b Reactive power at each node during load increasing with a controller

#### IV.B. Simulation Case 2

In the second case the active power consumed in the entire system is decreased by 50% and the reactive power consumed is calculated based on the randomized power factor. Similar to the first simulation case, the voltage profile is depicted in Figure (5a-5b). For the case with control the voltage has a maximum of 0.5% error.

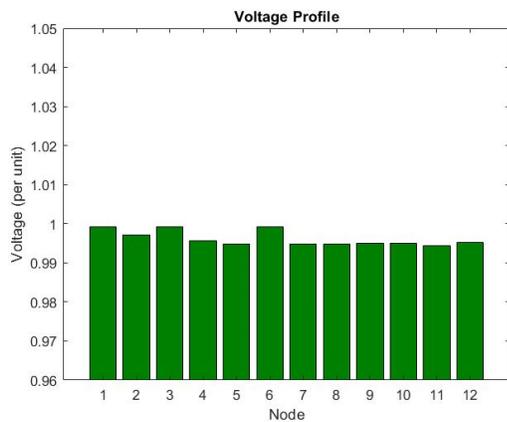


Figure 5a Voltage profile during load decrease with no controller

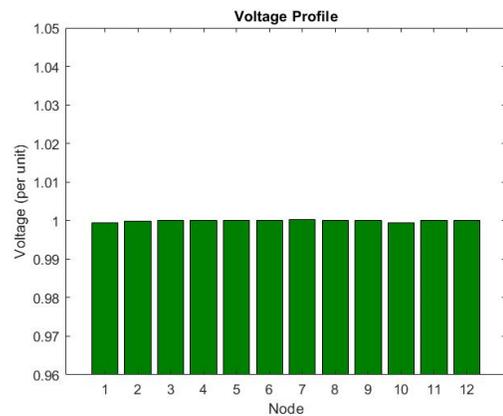


Figure 5b Voltage profile during load decrease with a controller

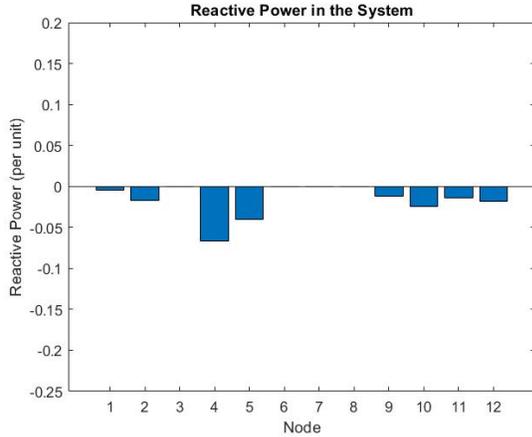


Figure 6a Reactive power at each node during load decreasing without a controller

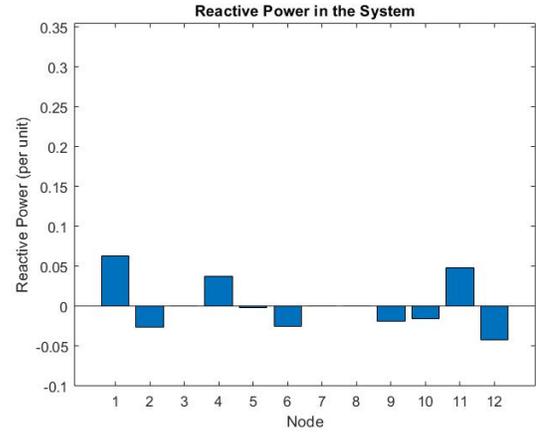


Figure 6b Reactive power at each node during load decreasing without a controller

In the second simulation case, the active power generated by the DERs is the same as that in the first simulation case. In case 2 both capacitors are off because there is a rise in voltage, as per Figure (6a-6b), the inverter at node 11 and 12 however injects and absorbs reactive power in the distribution system. These results are generated using MATLAB, YALMIP toolbox and the gurobi optimization solver.

## V. Conclusion

This research formulates the optimization problem to dispatch the reactive power generated by inverters and utilize the mixed integer quadratic programming method to solve this problem. It can be used to determine the reactive power inverters can produce to eliminate voltage violations in the distribution system. The simulation cases in this research are used to validate how the proposed method mitigates voltage-dips and voltage swells using inverters and capacitor banks. This work can be used by system operators to schedule proper dispatch and improve power quality. The algorithm can also be used within a centralized control environment to provide the reactive power references for local controllers within the DER. The proposed computational method is validated by demonstrating how the voltage profile remains within the appropriate tolerance given by standard 1547 [1]. VVC for a single-phase distribution system that contains DER at multiple nodes has been included in this research. Although the fixed capacitor bank included in the system is economical, its rigid reactive power supply is not ideal for a dynamic system. An automatic capacitor bank can keep a constant high-power factor under fluctuating loads. A limitation of this work is that it does not evaluate the behaviour of other VVC devices such as transformer load tap changer and voltage regulators. This model also ignores the power losses. The future work can include the application of the proposed method to the Dominion Energy system. Also, instead of the LinDistFlow model, a linear approximation of the multiphase branch model will be studied. This model can be applied to an unbalanced three phase system.

## BIBLIOGRAPHY

- [1] IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces," in *IEEE Std 1547-2018 (Revision of IEEE Std 1547-2003)*, vol., no., pp.1-138, 6 April 2018
- [2] M. Hadi Amini, S. Bahrami, F. Kamyab, S. Mishra, R. Jaddivada, K. Boroojeni, P. Weng, and Y. Xu, Chapter 6 - Decomposition Methods for Distributed Optimal Power Flow: Panorama and Case Studies of the DC Model, Classical and Recent Aspects of Power System Optimization, Academic Press 2018, pp. 137-155.
- [3] M. Farivar and S. Low, 'Branch flow model: Relaxations and convexification – Part I,' *IEEE Trans. on Power Systems*, Vol. 28, No. 3, Aug. 2013.
- [4] M. Baran and F. Wu, 'Optimal sizing of capacitors on a radial distribution system,' *IEEE Trans. on Power Delivery*, Vol. 4, No. 1, Jan. 1989.
- [5] V. Kekatos, L. Zhang, G. Giannakis and R. Baldick, "Voltage regulation algorithms for multiphase power distribution grids," *2016 IEEE Power and Energy Society General Meeting (PESGM)*, Boston, MA, 2016.
- [6] K. S. Ayyagari, N. Gatsis, A. F. Taha, and B. Dong, "On Static and Adaptive Policies for Chance-Constrained Voltage Regulation," *2018 52nd Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, 2018, pp.
- [7] H.-G. Yeh, D. F. Gayme, and S. H. Low, "Adaptive VAR control for distribution circuits with photovoltaic generators," *IEEE Trans. Power Syst.*, vol. 27, 2012, pp. 1656–1663.
- [8] A. Garg, M. Jalali, V. Kekatos, and N. Gatsis, "Kernel-Based learning for smart inverter control," *2018 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Anaheim, CA, USA, 2018, pp. 875-879
- [9] Z. Wang, H. Chen, J. Wang, and M. Begovic, "Inverter-less hybrid voltage/var control for distribution circuits with photovoltaic generators," in *IEEE Transactions on Smart Grid*, vol. 5, no. 6, Nov. 2014, pp. 2718-2728.
- [10] V. Kekatos, L. Zhang, G. B. Giannakis, and R. Baldick, "Fast localized voltage regulation in single-phase distribution grids," *2015 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, Miami, FL, 2015, pp. 725-730.
- [11] M. Farivar, L. Chen, and S. Low, "Equilibrium and dynamics of local voltage control in distribution systems," *52nd IEEE Conference on Decision and Control*, Florence, 2013, pp. 4329-4334.
- [12] (2018) Pecan Street Inc. Dataport. [Online]. Available: <https://dataport.cloud/>