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Assessing the Optimal Placement Algorithms of Phasor Measurements for State Estimation: a Critical Study

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SUMMARY

Power System State estimation utilizes the available measurements to estimate the state of the system, which is necessary for smart grids. Hence, obtaining an accurate and stable state estimator is the main objective to most of the meter placement studies that are related to the state estimation. However, Smart Grids started, recently, to employ advanced meters such as the Smart meters and the Phasor Measurement Units (PMUs). However, a clear majority of the available PMUs placement algorithm discards the quality of the state estimation solution and focus only on achieving a completely observable system. This leads to creating non-realistic meter configuration as the power systems have already their own measurement set, and hence, the optimal placement algorithms should concern the normal configuration of the power networks before suggesting a new scheme. This paper presents a critical study and investigates the validity of the existing optimal placement methods and their response to the requirements of the state estimation solution. The paper provides suggestions to the practical placement strategies.

KEYWORDS

Meter Placement, Phasor Measurement Units, Observability, Power System State Estimation.

INTRODUCTION

The operation of the power system State Estimation (SE) is employed by the Energy Management System (EMS) as a part of the wide-area monitoring and control activities [1]. The state of a power system includes the voltage magnitudes and the phase angles of the all the system's buses [2]. The estimated states facilitate the decisions of the monitors regarding the different contingency events. However, the state estimator using the iterative Weighted Least Square (WLS) problem that is sensitive to round-off errors and it is affected by the types and the numbers of the measurements. Hence, an accurate SE needs optimal meter placement strategies.

The state estimator utilizes the available measurements set for producing the state estimation solution. If the available measurements are sufficient to estimate the state of each bus, the system is declared to be observable [3]. The conventional measurements set can be measured by the power injection (active/reactive) meters, the power flow (active/reactive) meters, the voltage meters, and the current magnitude meters. The state estimator gathered the measurements from different locations in the power system via SCADA system. However, the conventional meters are not efficient regarding the robust monitoring, the contingency conditions, and the dynamic state estimation. Hence, recently, the Phasor Measurement Units (PMUs) became highly demanded by the power systems applications, especially, the Smart Grids applications [4]. In addition to the EMS, the Distribution Management System (DMS) became necessary for the active distribution grids. The PMU provides accurate and instantaneous measurements using the GPS. However, optimal placement strategies need to be achieved for determining the optimal numbers and locations of those expensive devices, i.e., the Optimal PMU Placement (OPP) [5].

The problem of OPP has been investigated in an enormous number of articles in terms of several power systems applications [5], [6]. However, most of the available OPP studies have no concerns about the quality of the SE solution; even that articles hold the "state estimation" in their titles [7]. Moreover, they do not consider the available conventional measurements in the power system. Rather, most OPP studies try to re-design the configuration of the meter from scratches.

This paper is part of a multi-stage research that focuses on the discrepancy between implementing robust and accurate power system state estimators for the modern distribution systems and the studies that discard the multi-objective problem of the PMUs placement. Accordingly, this paper presents a critical review and comparisons to available placement algorithms and their optimization constraint. Moreover, the mathematical background of SE solution, the derivation of the OPP algorithm, and the different objective functions are presented in this paper.

FORMULATION OF WLS STATE ESTIMATOR

The conventional nonlinear WLS state estimator is based on the following formula [8]:

$$z = h(x) + \varepsilon \quad (1)$$

where z is the measurement vector, x is the state vector, $h(x)$ is a nonlinear vector function, ε is the error vector.

The solution of (1) is the WLS estimation of x . The aim is to obtain the value of estimated state x that minimize $J(x)$. Hence, the objective function can be rewritten as follows [9-11]:

$$J(x) = (z - H(x))^T W (z - H(x))$$

The solution is subjected to the first-order optimality condition, which is:

$$H_x^T W [z - H(x)] = 0$$

where H_x is the measurements Jacobian matrix. However, the H_x is used in early in the linear state estimation. In linear (non-iterative) the Jacobian matrix is used directly in (1) instead of $h(x)$. That means all the collected measurements are stable and as accurate as the PMU and not a time varying. This is the first objection that turns the OPP to be a non-realistic optimization approach [12]. Thus, linear state estimation sacrifices the accuracy [12],[13]. Nevertheless, the final form of both the state estimators is the same:

$$G(x^k) \Delta x^k = H^T(x^k) W \Delta z^k \quad (2)$$

where x^k refers to the value of state x at the k th iteration and $G(x^k)$ is known as the gain matrix for the k th iteration. The gain matrix should be square, positive definite and symmetrical for observable systems [11].

OBJECTIVES OF OPTIMAL PLACEMENT OF PMU

Utilizing the advance PMUs with their communication channels need to be justified. Hence, it is agreed that the OPP is a multi-objective problem and, thus more than one constraint should be considered in the optimization algorithm [15]. This section focuses on the common objectives of the OPP problem relevant to the state estimation.

Power System Observability

Achieving a complete observability for the power systems is the main objective of any meter's placement algorithm. However, regarding the state estimation problem, there are two types of observability can be implemented, the numerical observability and the topological observability [16]. The numerical observability of a power system is achieved when the Jacobian matrix has a full rank, which is equal to the state vector. However, this analysis might complicate the placement algorithm due to the rank checking that requires building the measurements Jacobian matrix.

On the other hand, the most efficient approaches of observability assessment is the topological one since it is a graph-based process. Hence, the observability status can be identified without the full Jacobian matrix. The configuration of the power system and its connectivity reflect the necessary PMUs configuration. From the Graph-Theory perspective, if all the buses and the lines of a power system are included in a graph $G(N, B)$ such that N is the graph nodes, and B is the graph branches, the power system is declared to be observable topologically [17].

On the other hand, the topological observability can be achieved using a minimal number of PMUs when using the electrical circuits' laws. The Ohm's law and Kirchhoff's laws (KCL and KVL) can substitute many missing measurements, and thereby. That means, the vertices and the nodes of $G(N,B)$ can be observed either directly, or indirectly using the measured quantities of the adjacent buses/branches [19]. In this approach, the directly observable buses/branches belong to the nodes connected to the PMUs [18]. The indirectly observable buses/branches are those can be deduced from the available real-time measurements. The electrical circuits' laws as follows [19]:

Rule 1: Any bus can be observable if it is connected to a PMU bus. This is determined by the Ohm's and KCL laws.

Rule 2: Any branch between two observed buses is, in turn, observable (Ohm's law).

Rule 3: Any branch current can be computed using KCL law if all the incident branches are measured.

Regarding the optimal placement algorithms for a complete observability, the following procedure, which starts with connectivity matrix, is the most used in the literature:

$$c_{ij} = \begin{cases} 1 & , \text{ if } i=j \text{ or if } i \text{ and } j \text{ are connected} \\ 0 & , \text{ otherwise} \end{cases}$$

where C is the connectivity matrix that reflects the system's configuration. Then, the placement algorithm is as follows; this placement algorithm can be named as the connectivity-based optimization [19], [20]:

$$\min \sum_{i=1}^N p_i$$

$$\text{Subject to } C.P \geq \bar{R}$$

$$P = [p_1, p_2, \dots, p_n]^T$$

$$p_i \in \{0,1\}$$

where p is a decision vector whose entries are zero and ones based on the presence of PMUs and \bar{R} is a small natural value (such as one or two) that refers to the number of times each bus is observed by a PMU. For increasing the local redundancy, in the case of contingency, the value of \bar{R} can be two or three. The final solution of PMU placement problem is based on the number of ones that appear in the x vector. However, this placement approach aims to obtain bus/branch combinations that make the whole system monitored by the PMUs only.

The above constraint can be employed to express the required level of the measurements' redundancy which crucial for contingency analysis. This can be carried out easily by changing the value of $f(p)$ to be more than one, which refers to more redundant PMUs. However, the optimal solution should satisfy the $f(p)$ such that.

$$f(p) = N_{bus} - N_{obv} = 0$$

where N_{obv} refers to the observable buses which is eventually should be equal to the buses number.

Contingency and Redundancy

Placement of PMUs for contingency purposes seems to be irrelevant to the state estimation; yet, in the OPP it is associated with the redundancy rate which is, in turn, related to the measurements used for the state estimation. Any robust meter placement configuration needs to consider the case of meters failure, single power line outage, and outages of multiple lines. The available solution for meters and line outages is to increase the PMUs numbers or to changes their location in such a way that increases the measurements redundancy [21], [16,22]. The measurements redundancy aims to provide more meters for the same quantity. Increasing the local redundancy can be beneficial in the case of failure, but it contributes to the numerical instability of the SE solution [17]. Thus, this is the bad impact of most of the optimistic PMUs placement techniques. This process may be influential by increasing the required numbers of PMUs in the large-scale networks.

Requirements of Security and Bad Data Detection

This objective is essential for improving the quality of the SE solution. Nevertheless, the top-cited articles of PMUs placement rarely consider this aim as one of their fitness function constraints [5]. However, recently, the placement studies started considering the Bad Data Detection (BDD) and the cyber-attack as constraints in their objective functions [23]. Yet, the studies concern facilitating the BDD mainly come as incremental PMUs placement algorithms, i.e., not an explicit OPP problem.

COMPARATIVE CASE STUDIES

The authors explored the top-cited literature associated with the OPP problem in state estimation as those papers have attached the research topics of thousands of papers. The comparative study of this paper is based on the top 25 papers that have published under the title of "optimal phasor measurements placements for state estimation" and other similar titles. According to the authors' investigations, around 5000 articles are based on those 25 papers. Thus, they influence the trends of the OPP studies dramatically.

The main observation is that all the most cited papers are interested in the observability problem as the objective function of the PMU placement algorithms. Table I illustrates the major features of the articles are related to this study. However, different types of observability (numerical and topological) have been investigated in those papers. Therefore, other placement objectives that are discussed in Section III have a marginal appearance in the pioneer papers.

On the other hand, Table II shows statistics about the algorithms that have been employed in the most cited papers. It is obvious that most of the paper uses the algorithm of section II. Although the search has been extended to include any PMU placement literature, the concentration of the most cited papers is one optimizing the PMUs configuration based on the most-connected bus/branch technique of section II.

TABLE. 1 Main Objectives of the Top-Cited Papers

Paper	Observability	Contingency	Quality and BDD
[17]	✓		
[24]	✓		
[25]	✓		
[26]	✓	✓	
[27]	✓		
[28]	✓	✓	
[29]	✓		

[30]	✓		✓
[31]	✓	✓	
[32]	✓		
[33]	✓		✓
[34]	✓	✓	
[35]	✓	✓	
[20]	✓		
[36]	✓	✓	
[37]	✓		
[21]	✓	✓	
[38]	✓		
[39]	✓		
[40]	✓	✓	✓
[41]	✓		
[42]	✓	✓	
[43]	✓	✓	
[44]	✓		
[45]	✓		

TABLE.2 Main Objectives of the Top-Cited Papers

Articles of the Connectivity-based optimization algorithm	Articles that consider other optimization constraints
[17], [24-45]	[17], [30], [33], [40]

CONCLUSION

This paper provides a critical review to the studies of the optimal PMU placement in terms of the state estimation problem. Moreover, comparisons of the characteristics of the placement algorithms of the most cited papers are presented in this paper. The comparisons show that the common optimistic philosophy of the OPP problem cannot respond efficiently to the requirements of the grown smart grids. Thus, incremental placement strategies are recommended for more comprehensive placement algorithms.

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