Analytical and Data-Driven Methods for Determining Wind Farm Collector System Equivalent Impedance

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SUMMARY

The wind farm collector system connects multiple wind turbines together and evacuates power to the grid point of interconnection. For large wind farms, the collector system may consist of many cables in complex topologies. The impedances of these collector cables are usually aggregated into a single equivalent collector impedance which is used to model the wind farm in the larger power system analyses. This paper proposes two methods to simplify the determination of this equivalent impedance. The first uses a graph-theory based algorithm to calculate the equivalent impedance based on the industry-accepted NREL method [1]. The second uses data to model the impedance based on the rated power of the wind farm. This empirical method can be used to estimate the impedance before the collector system is fully designed. Both these methods help developers model and study their wind farms, improving interconnection capability for wind power.

KEYWORDS

Wind power, wind farm collector system, collector equivalent impedance, data-driven estimation, least-squares method, graph-theory application
I. Introduction

A key practical concern in the development of a wind farm is the design of an electrical collector system that consolidates power from the wind turbines and routes it to the substation where it is transmitted to the grid. The collector system comprises of medium voltage power cables of different sizes, typically connecting the turbines in several “daisy-chain” feeders. For large wind farms, this can create a complex network (Figure 1a). Instead of modelling this full network, it is often desirable to consider an equivalent collector connecting a single aggregate wind turbine to the substation, as shown in Figure 1b.

![Figure 1](image1.png)

**Figure 1** a) Full Wind Farm Connection Diagram; b) Aggregate Wind Farm Model

The aggregated wind farm model, which includes the aggregated representation of the collector system, is required for many applications. Some of these include representation of the wind farm in large system power flow cases and interconnection studies to evaluate stability issues due to low grid strength, sub-synchronous resonance, etc. The North American Electric Reliability Corporation (NERC) and grid codes around the world require validation of these aggregated plant models as well. The full detailed model of the wind farm is often not required. Rather, it is the industry standard to use an aggregated model to represent the behavior of the wind farm.

In this paper, two methods for quickly determining a collector system equivalent impedance are considered. A graph theory-based algorithm is presented to systematically calculate the impedance using an analytical method. Then a method is proposed to model the impedance based on the wind farm rated power using data from existing wind farms.

The National Renewable Energy Laboratory (NREL) has developed a collector system equivalencing method that aggregates the impedances of the medium voltages cables connecting each wind turbine [1]. The resulting equivalent system has one turbine generator connected through the equivalent impedance to the substation or point of interconnection (POI). The intent of the equivalencing method is for the losses in the equivalent system to be the same as those in the fully modelled system. Using this assumption, the series impedance of the collector system can be estimated as:
\[ Z_{eq} = \frac{V_{coll}^2}{S_{Loss}} \]  

(1)

In Equation (1), \( V_{coll} \) is the nominal voltage of the collector system, typically 34.5 kV in North America, and \( S_{Loss} \) is the apparent power lost between the turbines and the substation. Summing the losses in the system is not straightforward, since the losses on each cable section are dependent on the current carried by it. As individual turbines inject current along the feeder, the loading on the cables increases until the feeder reaches the substation. To simplify this analysis, the NREL authors assumed that the current injected by each turbine has the same magnitude and phase and that the voltage at each bus is close to 1 p.u.

The authors developed the following two equations to equivalence cables in different configurations. The first is for a feeder with \( n \) turbines in series where \( Z_i \) is the impedance between the \( i^{th} \) and \((i + 1)^{th}\) turbine starting from the end, as shown in Figure 2(a).

\[ Z_s = \frac{\sum_{i=1}^{n} i^2 z_i}{n^2} \]  

(2)

Once each series feeder is equivalenced, as shown in Figure 2(b), the set of feeders in parallel can be equivalenced by Equation (3). In this equation, \( n_i \) is the number of turbines on the \( i^{th} \) feeder and \( Z_i \) is the equivalent impedance of the \( i^{th} \) feeder.

\[ Z_p = \frac{\sum_{i=1}^{n} n_i^2 z_i}{(\sum_{i=1}^{n} n_i)^2} \]  

(3)

These equations can be used in combination for more complex topologies. For example, there are several places in the wind farm shown in Figure 1(a) where a single feeder branches off into two. In these cases, summing the current through each cable is complicated and can be time consuming.

The shunt susceptance for the collector system is calculated as the sum of all cable susceptances in the farm. This follows the assumption that each bus in the collector is at close to nominal voltage.

\[ B_{coll} = \sum_{i=1}^{N} B_i \]  

(4)

This method was validated by comparing power flow simulations using the complete and aggregate models. The current at specific locations in the network and the total losses throughout the collector system are compared and found to be within a few percent.
An alternative equivalencing method was presented by Su, Liu, Song and Xu based on the average voltage drop across the conductors on one feeder [2]. They proposed the following equation for \( n \) turbines in series:

\[
Z_S = \frac{\sum_{i=1}^{n}(\sum_{k=1}^{i}(z_k \sum_{j=k}^{n} p_j) p_i)}{\left(\sum_{i=1}^{n} p_i\right)^2} \tag{5}
\]

The authors showed that this method had less error in modelling the active and reactive power as compared to the NREL method. Accounting for changes in voltage throughout the wind farm allows for a more accurate loss calculation. However, the NREL method is preferred in this work since it is the industry standard method.

II. Equivalencing Methods

A. Detailed Analytical Method

The NREL equivalencing method provides a systematic framework for creating a simplified equivalent model. However, for large wind farms, working equations (2) and (3) by hand is time consuming and prone to error, especially when there are many feeders in complex topologies. The graph-theoretic algorithm proposed in this section can automatically calculate the power flow on each cable section. If it is assumed that all the turbines inject the same power, then this can directly provide the number of turbines connected to that particular section.

In [3], the authors presented a graph-theoretic algorithm where cable sizes are assigned based on maximum real power loading of cables and maximum real power generated by the wind turbines. Using a similar approach, in this paper a graph-theoretic framework of the wind plant collector system is considered where the wind turbines and junctions between different cable sections are modeled as nodes and the cables interconnecting them as branches or edges. Fig. 1(a) shows this visualization of a wind plant collector system. A graph with nodes and interconnected branches in which no direction is specified on the branches is called an undirected graph. If weights are not assigned on each branch then it is called an unweighted graph.

Typically, from an admittance matrix for a wind plant collector or by visually inspecting the collector system one-line diagram or design sheets, the undirected and unweighted graph of the collector system can be constructed. The next step is to convert this into a directed and weighted graph, where the direction would indicate the direction of power flow and weight would indicate the magnitude of power flow on each branch. This is where the algorithm described in this paper is applied, automatically computing the direction and magnitude of active power flow on the branches or cables. This in turn enables computation of the number of turbines connected to each cable section and allows automatic calculation of the equivalent impedance described in Equation (2). As can be observed by studying this equation, calculating \( i \), the number of turbines “upstream,” is the most computationally intensive step in the equivalencing method and is addressed by the algorithm described below.

Step 1) Two sets are initialized. Set 1 is the set of all turbine and junction nodes. Set 2 is the set of all branches. The total losses are set to zero.
Step 2) A node is selected from Set 1 such that it corresponds to only one branch in Set 2. This node is named Node A. The purpose of this step is to select one of the terminal nodes in the graph. Node A is denoted a “from node.” The node to which it is connected is denoted a “to node.” The power flow direction is from the “from node” to the “to node.” The flow on the cable is the power injected at the “from node” and the power injected at the “to node” location is incremented by the power flow on the cable. The flow on the cable is divided by the power injected by one turbine to get the number of turbines routing power through that cable. The ‘Temp’ variable is increased by the number of turbines squared multiplied by the series impedance of the branch. This represents the losses on that branch.

Step 3) Node A (“from node”) is deleted from Set 1 and the branch connecting the “from node” and “to node” is deleted from Set 2 thus resulting in a smaller dimension graph.

Step 4) Steps 2 and 3 are repeated until Set 1 is empty. Finally, Zs per Equation (2) is computed by dividing ‘Temp’ by square of ‘n’, i.e. no. of turbines of the feeder.

This algorithm has been further illustrated with a flowchart in Fig. 3.

Figure 3) Algorithm for computing power flow direction and magnitude on different cable branches, along with Zs per Equation (2).
B. Data-driven collector system impedance estimation

The algorithm presented above is able to systematically equivalence a collector system when each cable in the system is specified. However, cable data at this level of detail can be difficult to determine with good precision and requires the project to reach a certain stage in the development cycle before it is available. As an alternative, the collector system impedance can be modelled as a function of the wind farm’s active power rating. The farm rating is one of the few parameters known early on in a wind farm’s development.

The following equations are proposed to estimate the collector system impedance:

\[
\begin{align*}
\hat{R} &= \alpha_R P + \beta_R \frac{1}{P} \\
\hat{X} &= \alpha_X P + \beta_X \frac{1}{P} \\
\hat{B} &= \alpha_B P + \beta_B \frac{1}{P}
\end{align*}
\]  

(6)

Note that each polynomial has a linear and inverse power term. This follows the formulation in Equation (5), where the impedance has power in the numerator and denominator. The inverse term indicates that wind farms with lower MW rating will have higher impedance. Since such farms tend to have fewer feeders in parallel, the impedance can be larger. Also, the individual cables may be sized smaller in smaller farms, which could lead to larger impedances. The linear term prevents the impedances from falling to zero for farms with higher MW output. It also reflects the fact that large wind farms will span a large physical area, which will increase impedance.

To determine the coefficients in Equations (6), a set of 16 wind farms spanning from 40 MW to 300 MW was considered. These wind farms had all underground cables. An ordinary least squares approach was used to develop the coefficients in Table 1.

Table 1. Coefficients for Data-Driven Model

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$1.8094 \times 10^{-5}$</td>
<td>0.8378</td>
</tr>
<tr>
<td>$X$</td>
<td>$4.1655 \times 10^{-5}$</td>
<td>1.2695</td>
</tr>
<tr>
<td>$B$</td>
<td>$1.7363 \times 10^{-4}$</td>
<td>5.0640</td>
</tr>
</tbody>
</table>

These coefficients were used to model the impedances for 4 additional wind farms. Tables 2-4 show the equivalent impedances as computed by the analytical NREL method and the estimated values from the data-driven models in Equations (6). The last column in each table shows the error. Per unit values use the farm rating as the $S_{base}$.

Table 2. Collector System Resistance Fit

<table>
<thead>
<tr>
<th>$P$ [MW]</th>
<th>$X$ [p.u.]</th>
<th>$\hat{X}$ [p.u.]</th>
<th>$\Delta$ [p.u.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>0.0111</td>
<td>0.0152</td>
<td>0.0042</td>
</tr>
<tr>
<td>97</td>
<td>0.0165</td>
<td>0.0104</td>
<td>-0.0061</td>
</tr>
<tr>
<td>136</td>
<td>0.0069</td>
<td>0.0086</td>
<td>0.0017</td>
</tr>
<tr>
<td>258</td>
<td>0.0080</td>
<td>0.0079</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>
Table 3. Collector System Reactance Fit

<table>
<thead>
<tr>
<th>( P \text{ [MW]} )</th>
<th>( R \text{ [p.u.]} )</th>
<th>( \tilde{R} \text{ [p.u.]} )</th>
<th>( \Delta \text{ [p.u.]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>0.0263</td>
<td>0.0239</td>
<td>-0.0024</td>
</tr>
<tr>
<td>97</td>
<td>0.0370</td>
<td>0.0172</td>
<td>-0.0199</td>
</tr>
<tr>
<td>136</td>
<td>0.0081</td>
<td>0.0150</td>
<td>0.0069</td>
</tr>
<tr>
<td>258</td>
<td>0.0113</td>
<td>0.0157</td>
<td>0.0043</td>
</tr>
</tbody>
</table>

Table 4. Collector System Susceptance Fit

<table>
<thead>
<tr>
<th>( P \text{ [MW]} )</th>
<th>( B \text{ [p.u.]} )</th>
<th>( \tilde{B} \text{ [p.u.]} )</th>
<th>( \Delta \text{ [p.u.]} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>0.0993</td>
<td>0.0958</td>
<td>-0.0035</td>
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<tr>
<td>97</td>
<td>0.0677</td>
<td>0.0692</td>
<td>0.0014</td>
</tr>
<tr>
<td>136</td>
<td>0.0363</td>
<td>0.0609</td>
<td>0.0246</td>
</tr>
<tr>
<td>258</td>
<td>0.0584</td>
<td>0.0644</td>
<td>0.0060</td>
</tr>
</tbody>
</table>

As can be seen from the tables, the data-driven model provides good estimates of the impedances and susceptance. In some cases, the models are able to fit the impedances and susceptance up to 3 decimal places. They also follow the trend in \( P \), with larger values for small farms that decrease and level off as farm rating increases. Note that for some of the cases, the error is larger. This is observed mainly with the resistances. It is well-known that resistances are much smaller in magnitude compared to the reactances, hence are harder to measure and to match. The susceptance is also difficult to measure and match because it depends on local soil conditions. Using data from more wind farms could improve the performance of equations of these forms. A weighting scheme or more complex regression technique may be able to improve the fits as well. The proposed methodology shows the effectiveness of available data to train a simple model that can quickly estimate the collector equivalent for study purposes.

### III. Conclusion

The methods proposed here ease the process of determining collector system impedances both for the case when a detailed collector is known and for the case when the collector has yet to be designed. The NREL equivalencing method gives good results but can be cumbersome to implement by hand, especially for large wind farms. The algorithm presented here can be used to avoid calculation error and speed the process. A future study could implement the voltage-based method described in [2].

In general, the analytical method should produce more accurate results than the data-driven method, since it incorporates the actual cable information. However, the data-driven method can quickly provide estimated values early on in the wind farm development while still taking into account the differences in large and small wind farms. Such estimates are often crucial in modeling the wind plant for integration studies even before the wind plant goes online. While fitting the empirical models with a larger set of datapoints would improve the results, even with the data set used in this paper, the advantages of the data driven model can be clearly observed, with impedances for some cases matching the analytical model up to three decimal places.
BIBLIOGRAPHY

