21, rue d'Artois, F-75008 PARIS
http : //www.cigre.org

CIGRE US National Committee
2018 Grid of the Future Symposium

Generalized Approach for Volt-VAr-Control through Integration of DERs with Traditional Methods

D. RAZ
mPrest Systems
HIT – Holon Institute of Technology
Israel

A. DALIOT
mPrest Systems
Israel

SUMMARY

The paper presents a generalized holistic approach for integrating Volt-VAr control (VVC) resources such as capacitors banks and transformer tap changers, with Distributed Energy Resources (DERs), such as photovoltaic farms and batteries, in order to achieve various VVC target functions such as zero VAr flow at the transformer or minimal grid losses. The approach enables the operator to change the target function and takes into consideration the constraints of the system such as keeping the voltage within allowable limits. The paper introduces a method for reduction of complex distributions system into a simplified form, which enables the fast execution of the power flow calculations. Examples supporting this holistic approach are shown, demonstrating the capabilities of the method.

KEYWORDS

Algorithm, Forward-Backward Algorithm, Power flow, Reactive Power, Voltage Control, Volt-VAR control.

arield@mprest.com

Introduction

Traditionally, distribution systems include VVC devices that target to maintain the voltage within allowable limits, as required by the grid code or by power quality standards such as EN 60150 [1-2]. These voltage limits must be maintained at all circumstances and failure to meet these limits may result in malfunction, damage to electrical equipment or regulatory sanctions and fines.

The load fluctuates during the day due to changing demand as a result of both energy use and energy supply by consumers as a function of many variables, such as the type of load, the type of infrastructure, geographic area, geographic climate, season, holidays etc. [3]. There are several ways to deal with this issue. There are indirect methods in which the VVC device is varying the reactive power (for example), which indirectly changes the voltage in the system. These indirect methods include distributed generation (DG), shunt reactors/capacitor banks, Static VAR Compensators (SVC), etc. Other devices are considered as direct methods, which directly change and affect the voltage. Among these devices, we can consider On-load Tap Changers (LTC), transformers, regulators and smart inverters [4].

Up to the last decade, telemetry and visibility of the electrical parameters in the distribution grid of power systems were limited. *Smart* measuring and communicating devices were integrated mostly in the transmission systems and up to the distribution stations. This means that there was not enough data regarding the components in the distribution system and therefore, the methods for controlling voltage levels were very basic [5]. The revolution of *smart grid* introduced smart equipment and edge technologies up to the distribution transformer and even inside the facilities and customer premises. These new technologies bring forth a vast amount of data and information to electricity utilities. On one hand, this amount of data increases the complexity of the analysis required for the decision making process, and on the other hand it allows for more sophisticated as well as more holistic methods for controlling the voltage levels. Moreover, this data and enhanced control capabilities allow for the control of various dependent electrical parameters such as the voltage and the reactive power known as Volt/VAr Control (VVC) [6-8]. The traditional operation for indirect voltage control is to measure the voltage at a preinstalled capacitor bank. When the voltage exceeds the permitted limits, the capacitor bank is connected or disconnected, in a manner that low voltage will result in connecting the capacitor bank while high voltage will result in disconnecting the capacitor from the system. The operator also has the ability to raise or lower the tap changer in the direct method.

The addition of VAr control may add other capabilities to the system such as reduced system losses, reduced transformer losses and control of the reactive power flow up to zero or even negative VAr flow. However, practical VVC applications must take into consideration some physical and economical aspects of the assets. For example, LTC as an electromechanical device has inherent transients and mechanical determination. Another example is the dependence of the solution on the cost function of the DG, the price of a photovoltaic generator is different from a gas turbine and so on [9]. There is some work done on the coordination and operation of VVC assets under various conditions. Some work is focused on the coordination of DGs and LTCs [10]. Others have developed the theory for coordinating the operations of switched capacitors and LTCs in a radial distribution system by approximating the problem as a constrained discrete quadratic optimization problem [11].

In this paper, a holistic method for VVC is presented. The method is based on control at a system level of the Volt/VAr assets in order to achieve one of seven possible target functions. The desired target function is a free choice of the operator or by the operation policy. The controlled assets are: 1) Capacitor Banks 2) LTC 3) Photovoltaic smart inverter[4]. The method can be extended easily to use other devices such as storage batteries and grid-edge

Volt/VAr devices. The paper describes an optimal power flow-based method for a holistic VVC, the various optimization functions and with the constraints and simulation results.
Operation of the DG consumes energy resources (some of them are expensive, such as those using higher cost fuels) and increases the operational age of the machine. The voltage control coordination is therefore necessary in the distribution network and has been a subject of interest in many research papers. Ma et al in [3] have used the hierarchical genetic algorithm (HGA) to optimise the power and voltage control system according to the number of control actions. In [4], an integrated voltage control called Coordinated Secondary Voltage Control (CSVC) has been proposed for controlling the LTC positions to ensure that voltage and loading constraints are satisfied during normal and emergency conditions. Another voltage regulation method in power distribution systems including DG systems has been developed in [5] through optimizing sending voltages using the Least Square method. Authors in [6] have coordinated the operations of switched capacitors and LTCs in a radial distribution system by approximating the problem as a constrained discrete quadratic optimization. In [7], a coordination method for operating DG and step voltage regulators, for improved voltage regulation, has been presented. However, none of these coordination methods has considered the priority/selectivity of different voltage regulating devices in the system with the presence of DG. This concept is incorporated in this paper for the purpose of increasing the effectiveness as well as reducing the operating cost of the control actions. The aim of this paper is to present a complete, sufficient and applicable voltage regulation and reactive power control method.

1. Reduced single line diagram for distribution system

The common structure of distribution systems is a radial or weakly meshed system such as the ones shown in Figure.1.

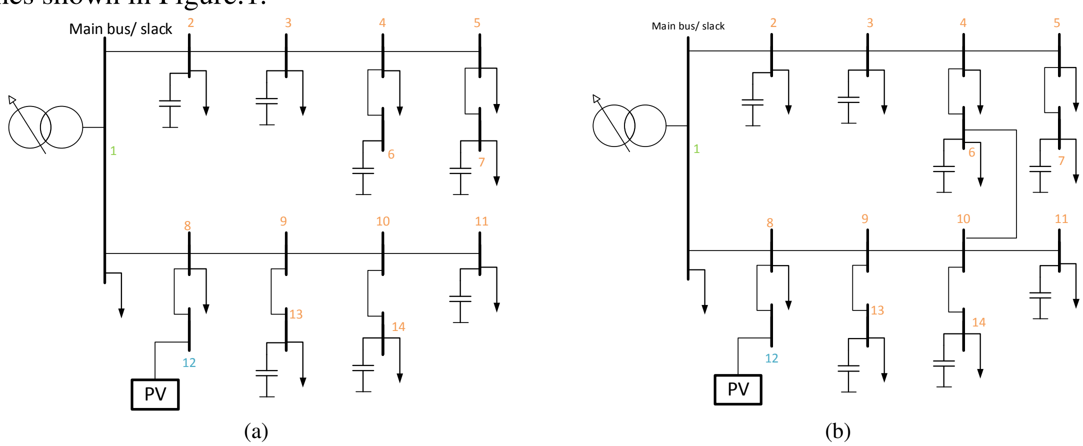


Figure 1: (a) an example of a radial system (b) and example of a semi or weakly meshed distribution system

The network in figure 1(a) is called a radial network. In graph theory, this is called a tree network of a directed acyclic graph. In figure 1(b), there is connection between bus 6 and 10 which results in a ring or loop in the distribution system. If the number of loops is small in comparison to the number of buses (nodes), this system is denoted weakly meshed (as opposed to heavily or fully meshed).
In general, distribution systems can be very complex in the sense that there are many lines (edges) and the number of loads can be very large. In Figure 2, such a system is shown, in a

graphic presentation of a typical distribution system, with various elements such as numerous distribution transformers, a few capacitor banks and DG. If all elements in this system would be monitored and measured then a full power flow could have been executed in order to compute all electrical parameters in the system.

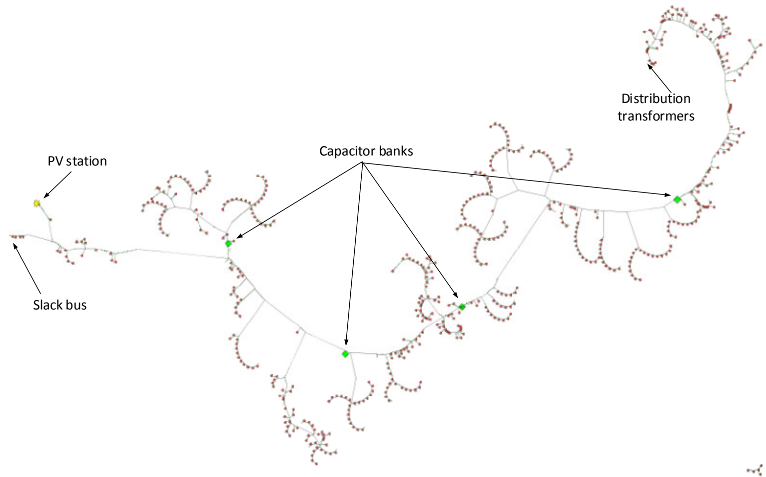


Figure 2: An example of a full distribution system

In most distribution systems, the number of measuring points is small in comparison to the complexity of the system. In many systems, measuring points are provided only for cardinal or dominant assets such as grid-scale solar farms, capacitor banks and the main bus (slack bus). This situation makes it difficult to compute the electrical parameters in the deep branches of the system. A solution is to transform the complex system to a reduced one-line diagram as shown in Figure 1. Since we are considering only radial networks (or weakly meshed that can be reduced to radial as will be shown later), we can look at the system as having a main feeder or a few main feeders that split into branches. Usually, measurement points will be located somewhere on the main line or very close to it. Therefore, we can use these points to be the main split points of the reduced diagram. In some cases, the system's topology may require a split into a branch at a different location. In these cases, there will be more than one bus in a branch (we define a branch as a diversion from the main feeder). An example of a reduced one-line diagram can be seen in Figure 1(a). It can be seen that branch points 2, 3 and 11 were modelled as a single bus branch, while other branch points were modelled as two bus branches. This means that the capacitor banks on branch points 2, 3 and 11 are modelled on the main feeder and all the loads and lines leading to the branch point are modelled as a single load. On the other busses, the measuring point of the capacitor bank/PV station is located further down the branch. It should be mentioned that since real measurements are typically available at the capacitor bank, in the case of dual-bus branches the electrical values of the main bus branch point must be estimated rather than measured. Naturally, this modelling method can be extended to include multiple layers of busses.

2. Power flow in distribution systems

Power flow is the most common and important and fundamental tool for the analysis of any power system in the operational as well as the planning stage. The power flow problem is the combination of the active and reactive powers. The equations for any bus k in the system can be formalized as,

$$
\begin{aligned}
P_k &= \mathrm{Re}\left\{V_k^*\left[V_k\sum_{j=0}^{n} y_{kj} - \sum_{j=1}^{n} y_{kj}V_j\right]\right\} \\
Q_k &= -\mathrm{Im}\left\{V_k^*\left[V_k\sum_{j=0}^{n} y_{kj} - \sum_{j=1}^{n} y_{kj}V_j\right]\right\},
\end{aligned}
\tag{1}
$$

Where P_k is the active power at bus k, Q_k is the reactive power at bus k and j is an index of all other buses that have a physical connection to bus k and y_{kj} is the kj^{th} entry of the admittance matrix that represents the topology. These equations are clearly nonlinear and *2(N-1)* equations are needed to describe a whole system. It is also known that the problem is difficult due being non-convex [12]. The most common methods to solve these problems are the Gauss-Seidel method and the more practical gradient based methods such as the Newton-Raphson algorithm and the decoupled Newton-Raphson method. It was already shown that these methods are initial state dependant and that convergence is not guaranteed, as the methods may all converge to a local minimum [13].

Moreover, distribution systems have some distinctive characteristics that require attention. These systems are radial with high ratio of reactance to resistance (X/R) due to the natural characteristics of short lines.

Most gradient methods such as Newton-Raphson are based on a Jacobian matrix, which is populated by the partial derivative of the network parameters. For the set of equations,

$$
\left.\begin{array}{ccc}
f_1(x_1, x_2, ..., x_N) & = & y_1 \\
f_2(x_1, x_2, ..., x_N) & = & y_2 \\
\vdots & \vdots & \vdots \\
f_N(x_1, x_2, ..., x_N) & = & y_N
\end{array}\right\}
\tag{2}
$$

A newton –Raphson form of representation will be,

$$
\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}
\tag{3}
$$

and the Jacobian matrix, J (constructed from J_1-J_4 in (3)) is something in the form of:

$$
J = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\
\frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\
\vdots & \vdots & & \vdots \\
\frac{\partial f_N}{\partial x_1} & \frac{\partial f_N}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N}
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{|V|}\frac{\partial P}{\partial \delta} & \frac{\partial P}{\partial |V|} \\
\frac{1}{|V|}\frac{\partial Q}{\partial \delta} & \frac{\partial Q}{\partial |V|}
\end{bmatrix}
\tag{4}
$$

In the case of distribution systems, this matrix is shown to be unstable and yields an ill-conditioned system [14]. Therefore, in the case of radial distribution systems other methods are preferred. In this paper, the backward-forward power flow method is used, as presented in [15]. This method was shown to be more stable and appropriate for this type of network. The method is comprised of two stages:

1) Transformation of weakly meshed distribution system to a radial one

This stage includes the scanning of the network in order to locate loops or buses with DG (the generation buses are treated in the same manner as loops).
The process turns a network such as shown in Figure 3(a) to a radial as in Figure 3(b).

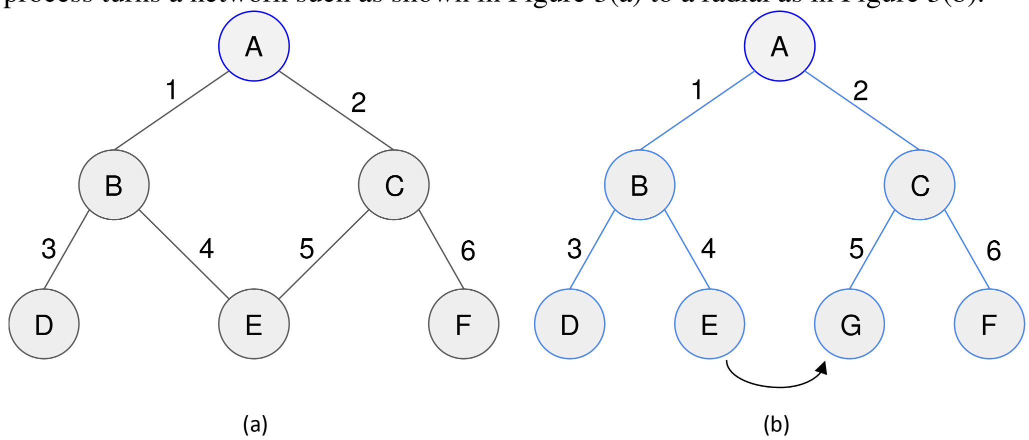


Figure 3: (a) a weakly meshed topology (b) radial topology after node E is split to E and G

There is a simple search algorithm that scans the nodes via the edges and when encounters the same node it divides it into two nodes, such as node E in figure 3(a), into nodes E and G in figure 3(b). Any power flow algorithm for radial systems can be executed now and only special constraints must be imposed on the new "virtual" nodes. The first is that joint power consumption of the original and the virtual nodes must be equal to the power consumption of the initial node consumption, namely (if E in figure 3(a) is the original node and E and G in figure 3(b) are the resultant nodes):

$$S_E\Big|_{fig.3(a)} = S_E + S_G\Big|_{fig.3(b)} \tag{5}$$

The second condition is that the voltage in these two nodes is equal to each other and to the original node. Namely,

$$\left.\begin{aligned} |V_E|_{fig3(a)} &= |V_E|_{fig3(b)} = |V_G|_{fig3(b)} \\ \angle\theta_E\Big|_{fig3(a)} &= \angle\theta_E\Big|_{fig3(b)} = \angle\theta_G\Big|_{fig3(b)} \end{aligned}\right\} \tag{6}$$

Under these constraints, a weakly meshed system can be treated as a radial system.

2) Backward-forward algorithm for radial systems

The power flow algorithm in this paper is based on the backward-forward algorithm presented in [15]. There is an initial assumption that the voltage in all buses is one power unit or 1pu. Then series of steps are executed as described ahead.

- *backward-step:* calculate the branch's currents as the sum of the currents in the nodes in lower layers of the system. The current in each branch is:

$$I(B_k) = \sum_{v_j} \frac{S_j}{v_j} \cdot U(k-j) \Big| U(k-j) = \begin{cases} 0 & if \ \ j>k \\ 1 & otherwise \end{cases}, \tag{7}$$

where $I(B_k)$ is the current in branch B_k that leads to node k, S_j is the power consumed by the node j, and v_j is the voltage in node j, as illustrated in figure 4.

- *forward step*: in each node voltage is calculated using the parent vertex's voltage and the branch voltage drop as follows:

$$v_k = v_{parent} - I(B_k) \cdot Z(B_k) \tag{8}$$

Where v_k and v_{parent} are the voltages at node k and its parent node, respectively, and $I(B_k)$ and $Z(B_k)$ are the current and impedance of branch B_k that leads from the parent vertex to the k vertex respectively, as illustrated in figure 4.

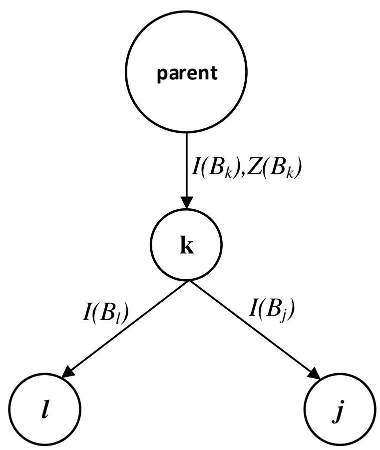


Figure 4: Backward and Forward steps illustration

Correction step: small corrections to the way the apparent power, *S*, is split between original and virtual vertices are performed.

The backward-forward steps are repeated until the following two stopping conditions are met:

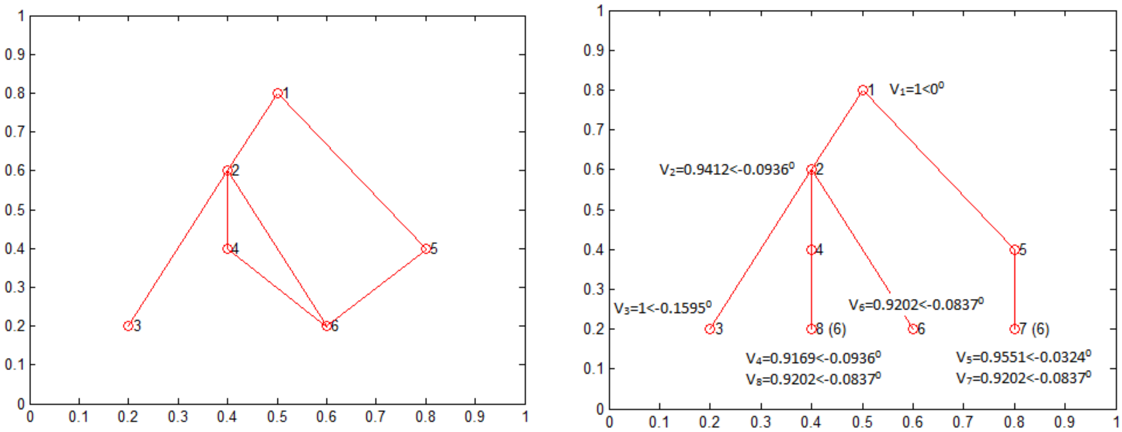
1. *Voltage condition:* the voltage at each node *k,* at the $(i+1)^{th}$ iteration needs to be equal up to small error, ε, (that can be defined as small as necessary) to the voltage calculated at the i^{th} iteration.

$$\begin{aligned} &\left| v_k^{(i+1)} - v_k^{(i)} \right| \le \varepsilon \\ &and \\ &\left| v_{virtual} - v_{original} \right| \le \varepsilon \quad in\ weakly\ meshed\ case \end{aligned} \tag{9}$$

2. *Power condition:* After calculating all voltages at the forward step, the powers of each node are calculated and are compared to the actual measured powers (which are the input of the algorithm).

$$\left.\begin{aligned} \left|P_k^{(i)} - P_k\right| \le \varepsilon \\ \left|Q_k^{(i)} - Q_k\right| \le \varepsilon \end{aligned}\right\} \tag{10}$$

We applied the power flow method on a semi-meshed network that was presented in [15].



(a) (b)

Figure 5: simulation results of a 6-node, weakly meshed network (a) the original network (b) the solution after splitting of node 6 and the calculation results

As presented in figure 5, the network was successfully converted to a radial network, and the power flow produced desirable results in comparison to the results in [15].

3. Holistic method for Volt-VAr control with DERs

The advantage of the backward-forward algorithm is its stability and rigidness when convergence is considered, especially in comparison to gradient-based methods. The reduced one-line diagram is a simple way of minimizing the network into a radial network with a limited amount of elements. These above-mentioned facts enable the use of methods that can lead to global optimization in the use of DERs and the VVC elements and combine them to a whole system. Minimization of the elements makes it realistic to use brute force methods as well as dynamic programing methods as described in [12], and still get results at real time.

The power flow algorithm can now be a part of a more general algorithm for optimizing the DERs such as photovoltaic farms connected via smart inverters and the VVC in order to achieve a certain target function as desired by the DSO. Most, VVC assets today are locally controlled. This means that when a voltage drops below a threshold value at a certain measurement point in the system, a local controller will connect a local capacitor to increase the reactive power at that point and attain a voltage rise to acceptable limits. The addition of more measuring points and remote controlling of all VVC elements can lead to a different solution that will imply better use of the system.

Various operation modes can be proposed as target functions for the algorithm. Among them are:

- Flat voltage profile: get the best voltage profile and closest to the nominal of the system.
- Conservation Voltage Reduction (CVR): keep the voltage in acceptable limits but aim the voltage profile to be closest to the minimal allowed values.
- Minimum asset operation: maintain voltage in acceptable limits by using minimum VVC assets to attain it.
- Zero VAr flow: operate VVC assets in a manner that the VAr value at the main bus (Slack) is closest to zero
- Designated VAr flow: operates VVC assets to reach the closest value to a predefined VAr value at the slack bus, including negative VAr.
- Predefined power factor: operates VVC assets in order to reach a predefined power factor at the slack bus, as may be required by regulations.
- Minimum power losses: operate VVC assets in order to minimize the total power losses of the system.

The algorithm in figure 6 is just a general overview of the method. When the VVC function is activated, it first gets all the necessary static and dynamic data from the database. Static data includes the connectivity of the one-line diagram, the impedance and length of the lines and the electrical parameters and rating of relevant assets. Dynamic data includes voltage and power measurements and the status of each asset. The optimal power flow function is executed after the operator chose the desired target function.

The algorithm creates a solution space that includes all possible solution for the target function and then chooses the best solution.

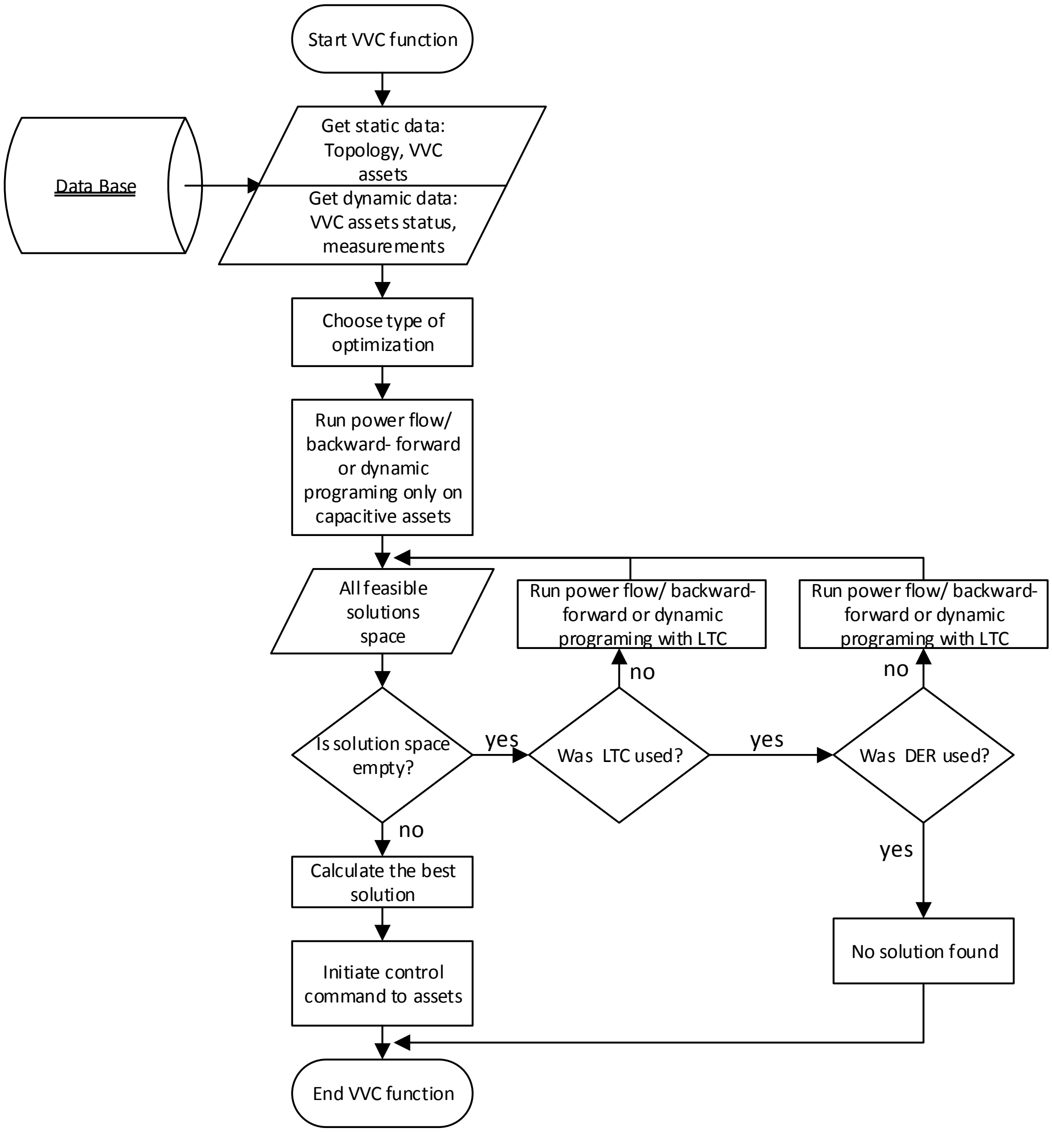


Figure 6: a flow diagram of the holistic VVC algorithm that include capacitors, LTC and DERs

The algorithm first looks for a solution including capacitors only. If the solution space is empty, it means that no solution is adequate for the target function and the calculation is repeated with the addition of LTC then with the addition of smart inverters. If no solution is achieved in all cases, the algorithm announces that there is no solution, in all other cases it will initiate a control status to all the VVC controlled elements.

The description of the algorithm in figure 6 is very general and generic, and can be easily changed to accommodate any business rule that the operator imposes. For example the coordination of the assets can change from first-capacitors, second- LTC, third- DG to any desired order. Smart inverters are flexible since the power electronics can react instantaneously, moreover their continuous manner makes them a key role in future VVC control [17],[18].

4. Simulation results

The algorithm was implemented in MATLAB. The electrical parameters of the network were taken from an actual utility operating network. However, in order to emphasise the various target functions, the parameters of the tested network were exaggerated. This results in an expanded solution space so more options can be demonstrated.

The first example is performed on the part of the distribution system, for which the reduced one line diagram is shown in Figure 7. The system includes LTC, a grid scale solar farm connected to bus 7 and capacitor banks in buses 4, 5, 6, 8 and 9. The solar farm is assumed to generate constant active power with a fixed power factor and a smart inverter and a voltage regulator, which get the voltage to 1pu.

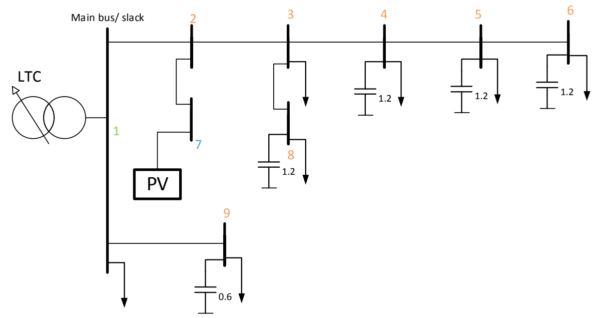


Figure 7: A 9-bus distribution system example

Figure 8 shows the result of executing the program with lower and upper limits of 0.9 and 1.1pu respectively. Note that for demonstration purposes, the voltage at the slack bus is kept 1pu rather than a real measured voltage. The resulting voltage profile is within the acceptable limits and therefore the system is healthy.

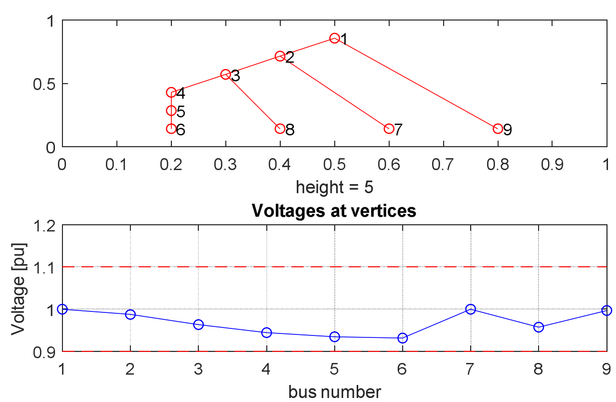
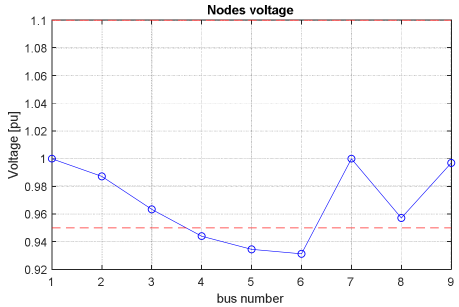
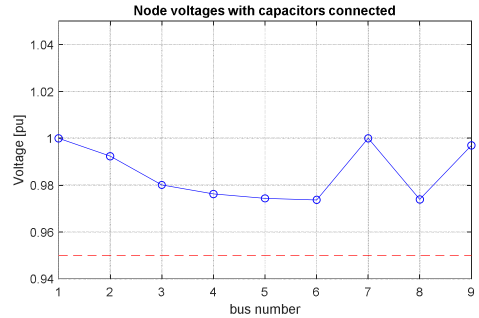


Figure 8: results of a healthy system

Lowering the voltage to 0.95pu, results in the voltage exceeding the lower limit in three nodes as seen in figure 9(a). In order to show the merit of the method, we first show the result from a localized solution. The voltage profile of figure 9(b) shows the resultant solution when the under voltage was detected by the controller of each capacitor at nodes 4, 5 and 6, resulting in the operation of these capacitors and resolving the under-voltage problem.



(a)



(b)

Figure 9: local treatment of under voltage (a) detection of under voltage (b) solution after inserting local capacitors

In figure 10, the result of the voltage profile of the previous local solution in blue (as in figure 9(b)) is presented in comparison to the voltage profile when an optimization program of CVR is performed.

The results of this optimization analysis suggests connecting capacitors 5 and 8 (instead of 4, 5 and 6 in the local control). It is clearly shown here that the CVR program yields a voltage profile that is within acceptable limits and closer to the lower limits. In this case, a 4.3% reduction in losses is also calculated.

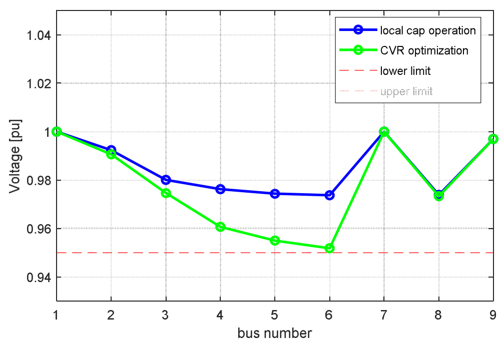


Figure 10: local control vs. CVR optimization

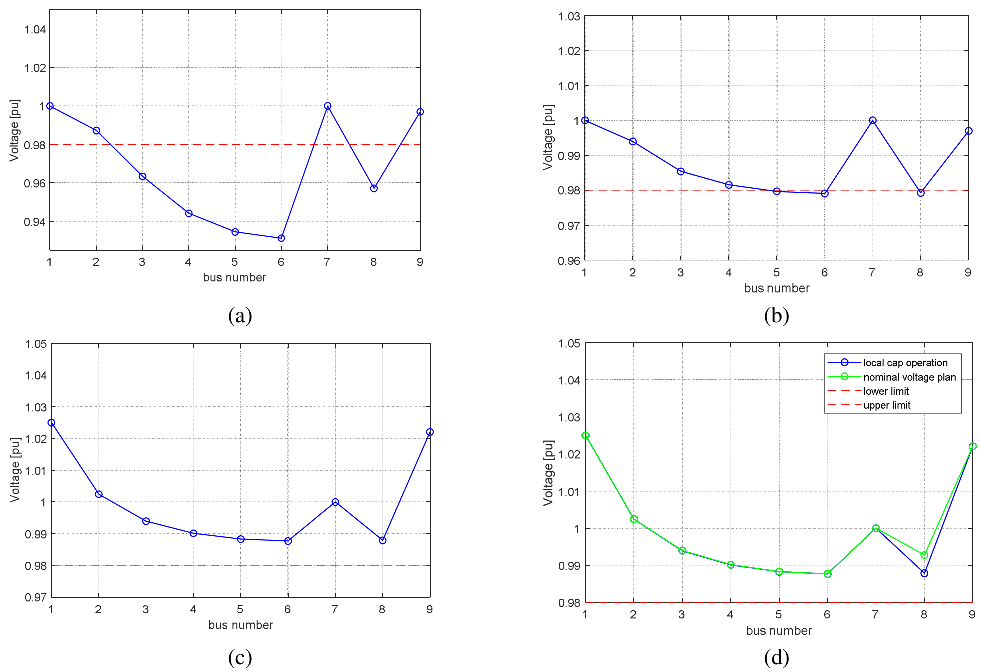


Figure 11: Tighter lower limit example (a) voltage profile with no VVC (b) voltage profile with local capacitors operation (c) addition of LTC (d) optimal solution with nominal voltage optimization

In figure 11, the lower limit was raised again to further stretch the example. The voltage before operation is shown in figure 11(a). In figure 11(b), the problem is not solved after operating the local capacitors at buses 4, 5, 6 and 8. The addition of a 2.5% rise in the LTC along with the capacitors solves the problem and the voltage profile is shown in figure 11(c). Next in figure 11(d), an optimization procedure of attaining nominal voltage is issued and the results show that the connection of capacitors 4, 6, 8 and 9 with a rise of 2.5% in the LTC will give a better solution by means of being close to nominal.
The last example in figure 12 shows the results of the simulation in the case of the 14-bus distribution system of figure 1(a). The results compare the voltage profile from local operation to that of an optimized operation with a target function of nominal voltage.

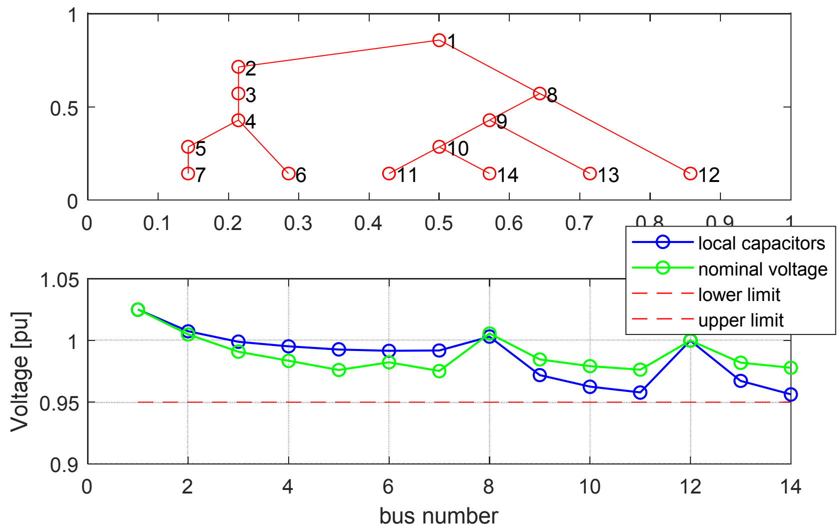


Figure 12: simulation results of the feeder in figure 1(a) with nominal voltage optimization vs. local operation

5. Conclusions

In this paper, a general holistic method of VVC is presented. The method includes a few stages. First, a reduced one-line diagram is generated in order to reduce a complex distribution system to a simplified radial or weakly meshed system. This reduced one-line diagram was shown to include the main measurement point or very distinctive buses of the system. In the case of a weakly meshed system, a procedure for transforming the network into radial system is shown. Then the paper has shown that the basis for the VVC optimization is a power flow calculation. The gradient-based methods were mentioned as unstable and therefore the backward-forward algorithm was chosen. The algorithm for VVC is then presented with the flexibility of the operator to choose a target function such as nominal voltage or CVR.
The paper then shows a few simulations on a 9-bus and 14-bus radial network and the results show that the operation of local capacitors yield a solution, but a more general view of the system can result in 1) better voltage profile 2) desired voltage profile 3) reduced VAR 4) reduced losses.

Acknowledgment

The authors would like to thank Dr. Yuval Beck for his contribution in simulating the results.

BIBLIOGRAPHY

[1] C. Masetti "Revision of European Standard EN 50160 on power quality: Reasons and solutions. In Harmonics and Quality of Power" (ICHQP, 2010 14th International Conference on 2010 September 26 pages 1-7).

[2] Y. Yang, P. Enjeti, F. Blaabjerg, H.Wang "Wide-scale adoption of photovoltaic energy: Grid code modifications are explored in the distribution grid" (IEEE Industry Applications Magazine. Volume 21, Number 2 September 2015, pages 21-31).

[3] P. Kundur, NJ. Balu, MG. Lauby "Power system stability and control" (New York: McGraw-hill; 1994).

[4] TT. Hashim, A. Mohamed, H. Shareef "A review on voltage control methods for active distribution networks" (Electrical review, Volume 6, Jun 2012, pages: 304-12).

[5] A. Carvallo, J. Cooper "The advanced smart grid: Edge power driving sustainability" (Artech House, Mar 1, 2015)

[6] V. Borozan, ME. Baran, D. Novosel Integrated "Volt/Var control in distribution systems" (In Power Engineering Society Winter Meeting, Volume 3, 2001, pages. 1485-1490).

[7] Y. Liu, P. Zhang, X. Qiu "Optimal volt/var control in distribution systems"(International journal of electrical power & energy systems, volumen 24, number 4, May 2002 pages: 271-276).

[8] T. Nikman, M. Mayeripour, J. Olamaei, A. Arefi "An efficient hybrid evolutionary optimization algorithm for daily Volt/Var control at distribution system including DGs" (International Review of Electrical Engineering, volumen 3, number 3, pages:513-524).

[9] G.W. Kim, and K.Y. Lee “Coordination Control of ULTC Transformer and STATCOM Based on an Artificial Neural Network” (IEEE Transactions on Power Systems, Volume 20, Issue 2, May 2005 , pages: 580 –586).

[10] AD. Le, KM. Muttaqi, M. Negnevitsky, G. Ledwich "Response coordination of distributed generation and tap changers for voltage Support" (In Power Engineering Conference, 2007. AUPEC 2007. Australasian Universities, 2007, Dec 9 pages: 1-7).

[11] R. Baldick and F.F. Wu "Efficient Integer Optimization Algorithms for Optimal Coordination of Capacitors and Regulators” (IEEE Transactions on Power Systems, volume 5, Issue 3, Aug. 1990, pages: 805 – 812).

[12] A. Beck, Y. Beck, Y. Levron, A. Shtof, L. Tetruashvili "Globally solving a class of optimal power flow problems in radial networks by tree reduction" (Journal of Global Optimization, volume 3, Aug 2017, pages :1-30).

[13] KS. Pandya, SK. Joshi "A survey of optimal power flow methods" (Journal of Theoretical & Applied Information Technology, volumen 1, number 5. May 2008)

[14] SC. Tripathy, GD. Prasad, OP. Malik, GS. Hope "Load-flow solutions for ill-conditioned power systems by a Newton-like method" (IEEE Transactions on Power apparatus and Systems, volume 10, Oct 1982 pages: 3648-3657).

[15] G. X. Luo and A. Semlyen "Efficient load flow for large weakly Meshed networks" (IEEE Transactions on Power Systems, volume 5, 1990, pages: 1309–1316).

[16] TL. Baldwin, and S. A. Lewis "Distribution Load Flow Methods for Shipboard Power Systems" IEEE Transactions on Industry Applications, Vol. 40, No. 5, September/October 2004

[17] JW. Smith, W. Sunderman, R. Dugan, B. Seal "Smart inverter volt/var control functions for high penetration of PV on distribution systems" (In IEEE Power Systems Conference and Exposition (PSCE), 2011 IEEE/PES 2011 Mar 20 ,pp. 1-6).

[18] P. Jahangiri, DC. Aliprantis "Distributed Volt/VAr control by PV inverters" (IEEE Transactions on power systems, volume 28, number 3, 2013 pages: 3429-3439).