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Optimal Allocation of Small Low-voltage Controllable Capacitors

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SUMMARY

An analysis of small multi-stage capacitor and larger single-stage capacitors is analyzed in this paper. In order to site the low-voltage connected capacitors, a modified version of the Grainger/Lee capacitor placement methodology has been developed. The effectiveness of the developed method is tested using a nondominated sorting genetic algorithm II that optimizes the solutions on the circuit, optimizing both the power losses and voltage profile. Due to the network's dimension, and the small size of the capacitors and steps, the solution space is reduced by assuming the location of the smaller devices is limited to low-voltage portions of the circuit.

The results between the modified Grainger/Lee methodology and the genetic algorithm are compared for varying capacitor sizes. This is done with and without the existing capacitor banks as well as assuming different sizes of the low-voltage controlled reactive compensation. As expected, the overall reduction in losses is higher using smaller controllable capacitors; however, these solutions required a larger number of installed capacitors resulting in significantly higher costs to performance. Furthermore, the modified Grainger/Lee method is demonstrated to provide an effective estimate of optimal locations for any size capacitor across both medium and low-voltage levels.

KEYWORDS

Capacitor placement, reactive power compensation, volt-var optimization

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INTRODUCTION

In this work, an analysis is performed on the allocation of small low-voltage multi-stage capacitors (10 kvar rating controllable with 1 kvar steps) compared to standard medium-voltage single-stage capacitors for both power loss reduction and increased voltage flatness. A modification of the traditional Grainger/Lee capacitor placement methodology is applied to the standard EPRI Circuit 5 with and without existing capacitors present. The proposed modification is necessary to apply the method effectively to allocate the capacitors on both the medium and low-voltage branches of the circuit. It is found that losses are reduced if the var rating of the capacitor is less than twice that of its downstream reactive load, and its location can be optimized, in terms of power loss reduction, by placing the capacitor as far down stream as possible while still meeting this condition and considering the placement of capacitors on other branches. Thereby, not causing an upstream injection of reactive power greater than half the size of the capacitor.

In order to test the effectiveness of the modified Grainger/Lee method, a nondominated sorting genetic algorithm II (NSGA-II) is employed to optimize the solutions on the EPRI circuit, optimizing both the power losses and voltage flatness, measured as the root mean square deviation (RMSD) of all the busses' voltage from the mean value. However, due to the network's dimension, and the small size of the capacitors and steps, the solution space for the circuit is very large and impractical to manage without first reducing it. Running multiple scenarios with the modified Grainger/Lee methodology shows that the step-down transformers are most likely the best location for capacitor placement on low-voltage nodes, and that there is rarely more than one capacitor required downstream of the step-down transformer. This allows the genetic algorithm solution space to be reduced accordingly (only one capacitor per node and as far down as the low-side of the step-down transformers).

The results between the modified Grainger/Lee methodology and the genetic algorithm are compared both with and without previous capacitors installed in the feeder. Without previous capacitors installed, the modified Grainger/Lee method and the genetic algorithm provide the same power loss reduction. If the circuit has existing capacitors, the genetic algorithm outperforms the modified Grainger/Lee method. As expected, the smaller controllable capacitors outperform the larger capacitors, but their cost to performance is significantly higher and a larger number of capacitors are required.

In conclusion, the modified Grainger/Lee methodology is applicable for any size capacitor at any voltage level. Smaller capacitors can yield a greater reduction in power losses and provide a flatter voltage profile compared to their larger more traditional medium-voltage counter parts. However, the more traditional modified Grainger/Lee method can provide a good, but not optimal solution when capacitors are already present in the feeder compared to a genetic algorithm. Utilizing smaller capacitors are also more expensive ($/var) compared to larger ones, but they can be useful to solve local low voltage, power factor, or capacity issues.

MODIFIED GRAINGER/LEE METHOD

One benefit of installing capacitors is the reduction in distribution line losses. Capacitors provide reactive-power support reducing the amount of current in the line. Since active-power line losses are a function of the current squared, I^2R. Supplying reactive-power support closer to the load reduces the line currents upstream thereby reducing losses.

This section provides a brief overview of the Grainger/Lee methodology traditionally used for planning capacitor placement location in distribution feeders. Due to the small size of the capacitors being considered in this work (1-10 kvar compared to a more typical size of 300 kvar or more) a more detailed analysis is performed on optimizing the capacitor's location including secondary circuits for minimizing power losses. A modified version of the Grainger/Lee methodology is then proposed based on insights from the analysis.

Grainger/Lee Method

Grainger and Lee [1] provide an optimal yet simple method widely used by engineers for placing fixed capacitors on a circuit with any load profile, not just a uniformly distributed load. The reactive load profile of a circuit is utilized to place capacitors with the Grainger/Lee method. The basic idea is to place capacitor banks at points on the circuit where the reactive power equals one half of the capacitor's var rating. With this 1/2-kvar rule, the capacitor supplies half of its vars downstream, and half are sent upstream. The basic steps of this approach are [2]:

1. *Pick a size* — Choose a standard size capacitor.
2. *Locate the first bank* — Start from the end of the circuit. Locate the first capacitor at the point on the circuit where var flows on the line are equal to half of the capacitor var rating.
3. *Locate subsequent banks* — After a capacitor is placed, re-evaluate the var profile. Move upstream until the next point where the var flow equals half of the capacitor rating. Continue placing capacitors in this manner until no more locations meet the criteria.

However, the Grainger/Lee method only guarantees the optimal solution when applied to the primary feeder. Given the small size of the capacitors, it will be important to consider the influence of the low-voltage secondary in the allocation process.

Modified Grainger/Lee Method

More care and attention needs to be taken when placing smaller size capacitors, like the small multi-stage capacitors being discussed in this paper, since their optimal placement will most likely be on the low-voltage single-phase lines of the circuit. The main concern is to not place too many capacitors and back feed reactive power. A modified Grainger/Lee methodology has been developed and the steps are as follows:

1. Starting from the source, sort the *branches* by in a hierarchical/tree like structure by their reactive power consumption, Figure 1.A.
2. Starting from the *leaf* with the highest reactive power consumption of the *branch* with the highest downstream reactive power flow, continue to move to the next *parent branch* until the downstream reactive power flow at the *branch* is greater than or equal to one half of the capacitor rating.
3. Place the capacitor at the end of the *branch,* Figure 1.B and Figure 1.C.
4. Run a power flow of the circuit with the added capacitor.
5. If the upstream reactive power flow is greater than one half of the capacitor's rating at any *parent* of the *branch* where the capacitor has been placed, Figure 1.D, remove the capacitor.
6. Repeat steps 1-5 until the downstream reactive power flow at the source is less than the chosen capacitor size, or no other capacitors can be placed.

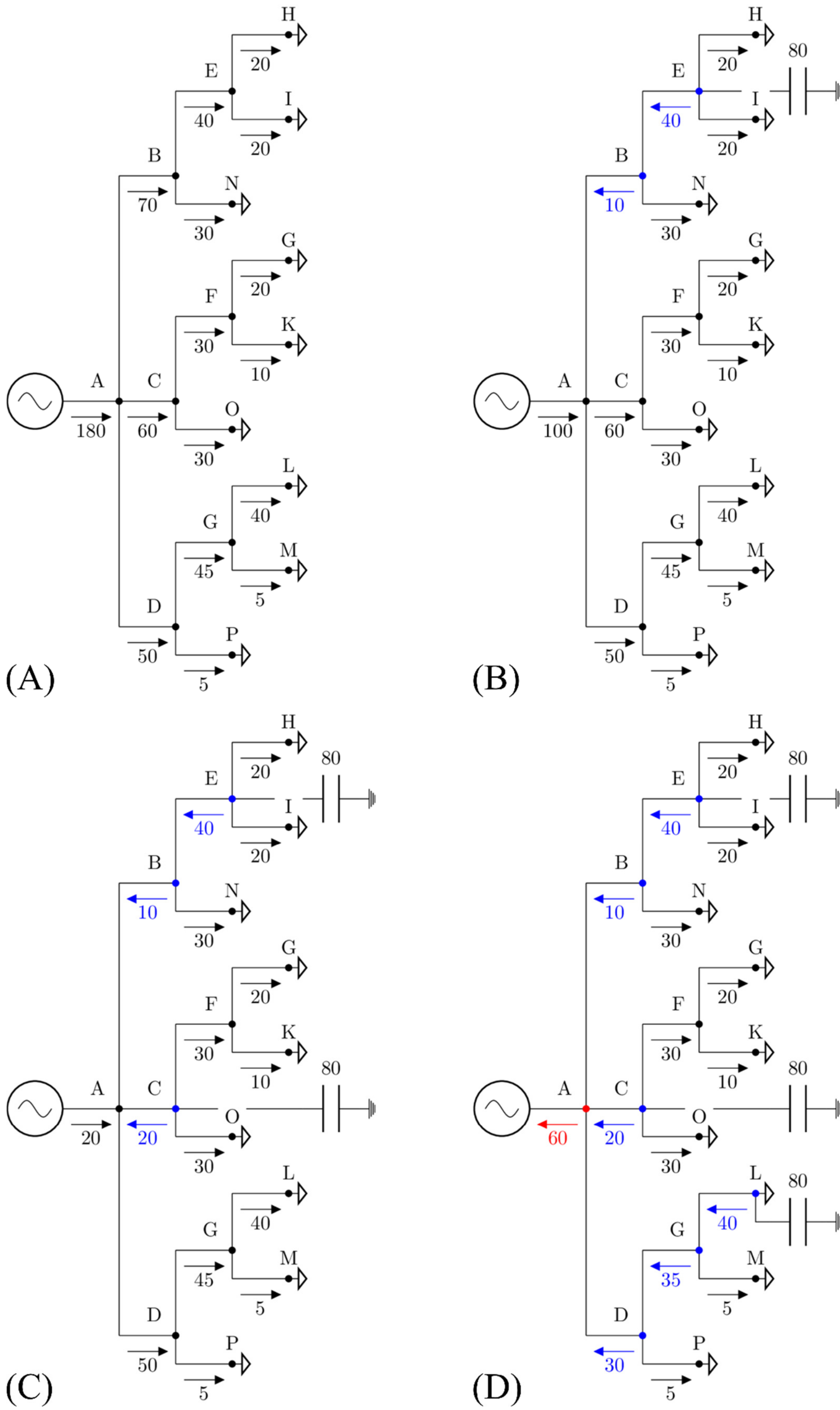

Figure 1: Example circuit for the modified Grainger/Lee method. A) shows the circuit with the reactive power flows before capacitors. B) shows the circuit with an 80 kvar capacitor added at node E and the updated reactive power flows in blue. C) shows the circuit with an 80 kvar capacitor added at node C and the updated reactive power flows in blue. D) shows the circuit with an 80 kvar capacitor added at node L and the updated reactive power flows in blue. The reactive power flow is now reversed at the source as shown in red.

Note, while the stopping conditions listed in step 6 of the modified Grainger/Lee method applies before placing the third capacitor at node L in Figure 1.D, it is still shown because step 5 may apply in a larger circuit with more sets of *parent* and *child* branches. Instead of node A connecting to the source bus, the circuit may represent a section of a much larger one.

Even though the Grainger/Lee and modified Grainger/Lee methods are simple iterative processes, they provide optimal capacitor placement when considering power losses. While [2] claims the Grainger/Lee method can optimize the placement of "new" capacitors on a circuit with existing capacitors, this can only be the case if the new capacitors can only be placed between the source and the existing capacitors. With smaller capacitors, the modified Grainger/Lee method can ensure "good," but not optimal placement of new capacitors anywhere on the circuit containing existing capacitors, with the inclusion of step 5 of the modified Grainger/Lee method. If there are larger reactive power loads on the circuit, the best location may be to just place the capacitor right at the load.

GENETIC ALGORITHM

To compare and validate the modified Grainger/Lee method, a NSGA-II [3] method is employed. The NSGA-II is an effective multi-objective algorithm that determines an accurate and well-spread Pareto front, which is useful for the subsequent decision-making process. The multi-objective formulation is necessary for finding trade-off solutions that can balance expenditures and benefits. By analyzing the set of optimal solutions provided by the NSGA, the distribution planner can find the most suited one for the allocation strategy of the capacitors.

The proposed procedure for the NSGA-II algorithm can be summarized with the following steps:

1. *Individuals' coding (allocation of capacitors)* – An individual represents a possible solution. Each individual is represented by a vector (chromosome) with a dimension (number of genes) equal to the number of candidate nodes, as depicted in Figure 2.

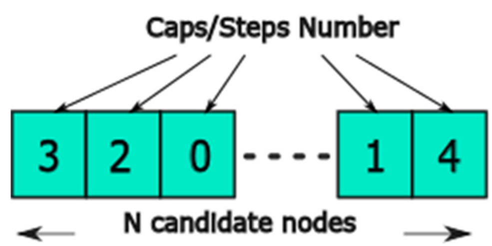

Figure 2: Example of a genetic algorithm chromosome.

   The genes can represent the number of capacitors that will be connected, or the number of steps for a given capacitor that will be used, in the corresponding node;
2. *Initial population* – A random generation of a population of individuals is created to start the optimization procedure;
3. *Evaluation of the objective functions* – The multi-objective optimization requires the definition of different objective functions (OFs), which are the contrasting objectives the planner would like to pursue. In this paper, three goals have been used. The first one is the minimization of the power losses in the distribution network. The second one is the maximization of the voltage profile flatness. The parameter used to evaluate the

flatness, is the RMSD of the per-unit voltage values measured at each node from the average per-unit voltage of the grid. The third is the total cost, which is directly proportional to the number of capacitors installed;

4. *Population's evolution* – By applying the definition of dominance between two individuals and introducing a crowding parameter within a set of non-dominated solutions (frontier), it is possible to rank the population and to use the classical genetic operators (selection, crossover and mutation) to evolve towards a new population, exploring the solutions' space [3];
5. *Stop criteria* – If the maximum number of generations is reached or if all the OFs stop to improve, the optimization procedure is stopped, otherwise it returns to step 3 with the new population.

RESULTS AND DISCUSSION

Application of the modified Grainger/Lee Method

The proposed modified Grainger/Lee method was tested on EPRI Circuit 5 [4]. Five different capacitor placement cases are considered:

CASE 0: Base case with no capacitors installed
CASE 1: 10 kvar single-phase low-voltage controllable capacitors with a 1 kvar step size
CASE 2: 10 kvar single-phase low-voltage capacitors
CASE 3: 100 kvar single-phase medium-voltage capacitors
CASE 4: 300 kvar single-phase medium-voltage capacitors

Two different scenarios are considered for each case. In the first scenario, no capacitors are installed in the circuit. In the second scenario, two 600 kvar three-phase capacitors, one 450 kvar three-phase capacitor, and one 300 kvar three-phase capacitor are already installed in the circuit. In both scenarios, loads are modeled at 90% of their nominal power. The results obtained with the application of the modified Grainger/Lee method are summarized in Table 1 and Table 2.

Table 1: Results of the modified Grainger/Lee method's application without feeder capacitors previously installed.

Case	Number of capacitors	Power losses (kW)	RMSD (pu)	Cost (k$)	Cost/capacity ($/kvar)	Cost/power saved ($/W)
CASE 0	0	223.49	0.0115	0		
CASE 1	581	202.93	0.0088	581	100.0	28.26
CASE 2	182	207.73	0.0084	182	100.0	11.55
CASE 3	24	212.23	0.0112	36	15.0	3.20
CASE 4	8	214.51	0.0112	32	13.3	3.56

Table 2: Results of the modified Grainger/Lee method's application with feeder capacitors previously installed.

Case	Number of capacitors	Power losses (kW)	RMSD (pu)	Cost (k$)	Cost/capacity ($/kvar)	Cost/power saved ($/W)
CASE 0	0	214.27	0.0111	0		
CASE 1	212	208.92	0.0096	212	100.0	39.63
CASE 2	74	210.32	0.0094	74	100.0	18.73
CASE 3	8	211.83	0.0093	12	15.0	4.92
CASE 4	3	212.85	0.0097	12	13.3	8.45

From Table 1, it is evident how the smaller capacitors allow a greater reduction in power losses when they are able to vary their step size. However, their cost is significantly higher and their cost efficiency, in terms of $/W, is worse than the larger capacitors. The same behavior is found in the second scenario, Table 2, where the feeder contains previously installed capacitors. Here, the power losses are higher than in the first scenario for all cases, and so is the cost efficiency. This is due to the proposed method not optimizing the total power-loss reduction when applied to a circuit already containing capacitors.

Varying Loading Levels

Only a single load level is considered in the previous analysis, 90% of the loads nominal power. However, load can vary greatly and is typically at 30-50% of its nominal power. Three different load levels are considered for placing capacitors:

Subcase A: the same as before with loads at 90% of their nominal power
Subcase B: the loads are at 70% of their nominal power
Subcase C: the loads are at 50% of their nominal power

The same analysis is performed as above considering no capacitors are previously installed on the circuit. With the capacitor placement determined, the losses are totaled over an 8760 study period and the results are shown in Table 3.

For CASE 1 the subcases A, B, and C are almost invariant. Remember, in CASE 1 the capacitors are controllable from 0-10 kvar, but they are only placed if at least 1 kvar of support is required. Even if a capacitor is set to provide only 1 kvar, the effect will be to place a capacitor on the low-voltage side of almost every step-down transformer, regardless if the load level is at 90% or 50%. This is true because if the placement is optimized for the losses when the load is at 50% its nominal, when the load increases the capacitors will increase their output. The same applies when the load is at 90% its nominal. However, now the capacitors will decrease their output, eventually down to zero. Therefore, the result will be approximately the same for both cases.

Table 3: Results of the modified Grainger/Lee method's application with varying load levels.

Case		Energy Saved (MWh)	Cost/energy saved (k$/MWh)
CASE 1	A	89	6.5
	B	88	6.4
	C	89	6.3
CASE 2	A	64	2.8
	B	52	1.9
	C	21	1.4
CASE 3	A	46	0.8
	B	42	0.7
	C	43	0.4
CASE 4	A	28	1.1
	B	24	1.0
	C	22	0.5

CASES 2-4 exhibit considerable differences for the three different subcases. Optimizing the allocation at higher load levels yields to more capacitors being installed. Leading the cost per MWh saved to be higher. If the capacitors are controllable, this ensures the power losses to be as low as possible, since it is will be possible to reduce or inhibit the action of the capacitors when they are not required. However, there may be an increase or smaller reduction in losses if the capacitors are not easily controllable. If the placement of the capacitors is optimized at lower load levels, when the flows are higher there is no way to further strengthen the action of the capacitors. The difference in the power losses for the different cases is much stronger for the 10 kvar capacitors because they have multiple steps, and therefore can be finely tuned to the current load level. In contrast, the larger 100 and 300 kvar capacitors must be fully switched in or out of service.

While the small controllable capacitors allow for a greater reduction in losses compared to more traditional larger capacitors, their cost efficiency is much lower and does not justify their adoption based on the power-loss reduction alone. In fact, the reduction in losses between the capacitors sizes was no larger than 5%.

Application of the NSGA-II Method

The development and application of the modified Grainger/Lee method indicates the best location for the capacitors to minimize losses and provides important indications on the benefits achievable by the allocation of the 10 kvar controllable capacitors compared to more traditional capacitors sizes.

As stated above, power losses alone are not enough to consider adopting smaller more controllable capacitors. However, capacitors can also be used for voltage support or to flatten the voltage profile. The Grainger/Lee method is designed to place capacitors based on power losses, nothing else. The NSGA-II method discussed in the previous section can be designed to optimize capacitor placement with any objective, including multiple objectives. In this case the NSGA-II method has been designed to minimize the power losses and optimize the voltage flatness in the most cost-effective manner.

The small size of the capacitors, particularly when considering them controllable with a 1 kvar step size, together with the high number of low-voltage nodes present in the circuit create a large solution space. Consequently, in order to effectively apply the genetic algorithm, it may be necessary to reduce the number of candidate nodes and to set a reasonable upper limit to the number of capacitors that can be installed in each node.

Considering the analysis of the modified Grainger/Lee method above, several conclusions from the results can be leveraged to help reduce the solution space. In particular:

- Because of the 1/2kvar rule, it is very seldom that one of the small multi-stage capacitors are placed anywhere but on the low side of the step-down transformers.
- Very seldom, even at peak load conditions, is it necessary to place multiple capacitors (of any size) at the same node.

The first observation allows for the available capacitor placement locations to be reduced to only the busses from the low side of the step-down transformers and up (smaller chromosomes). The second reduces the space further when each bus is limited to one capacitor only (reduction in the number of gene values).

The NSGA-II method with a multi-objective of power loss reduction and voltage flatness is run for both CASE 1 and CASE 2 presented above. The results from the NSGA-II method and those from the Grainger/Lee method are shown in Figure 3. The figure plots the Pareto front, i.e. the set of non-dominated solutions.

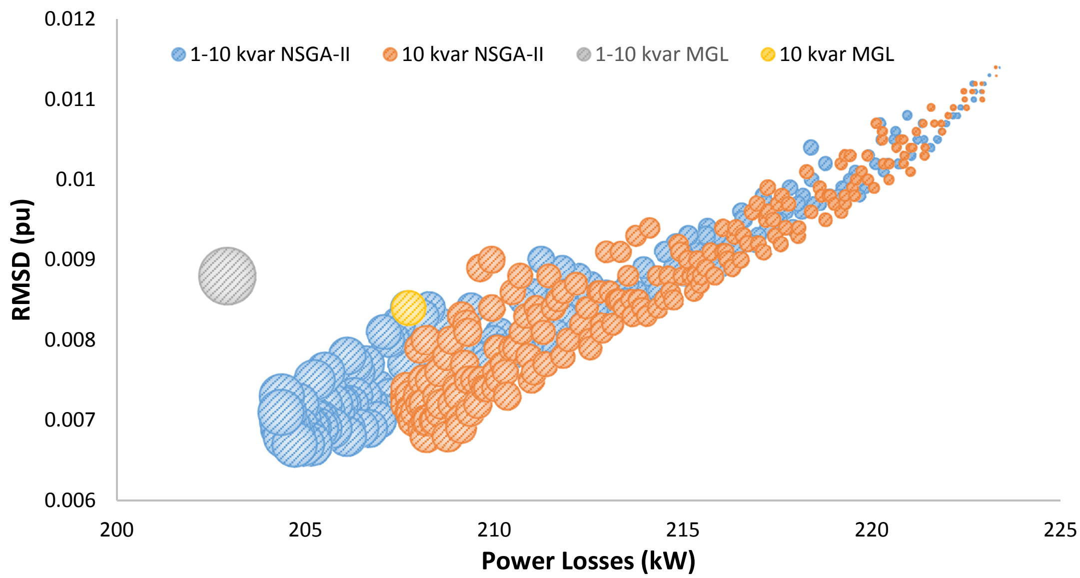

Figure 3: Pareto front of the solutions obtained with the NSGA-II and the modified Grainger/Lee (MGL) method, applied to EPRI Circuit 5 without feeder capacitors previously installed. The larger the bubble is the higher the cost ($0 – 192k).

As with the modified Grainger/Lee solution, the NSGA-II solutions yield better results with the controllable capacitors compared to those with a set value. But once again, the more optimal solution comes at a higher cost.

The more expensive solutions, i.e. a larger number of installed capacitors, are able to provide both lower losses and a flatter voltage profile for both CASE 1 and CASE 2. It also appears that the two objectives, minimizing power losses and RMSD, do not exhibit a contrasting behavior by the linearity of the two Pareto fronts. It can be licit to assume so, because the circuit does not contain any voltage regulation, and therefore the flattening of the voltage profile by the capacitors implies its rising. Because the loads in the circuit are modeled as constant impedances, the higher voltage increase the active and reactive demand of the feeder loads. However, in this case, the reduction in overall var flow by the reactive power compensation is determined to counter the resulting increased losses.

When comparing the results in Figure 3, the modified Grainger/Lee provides a solution very close if not slightly better than the NSGA-II solutions for loss reduction. The modified Grainger/Lee method works well for its objective. The solutions found via the NSGA-II algorithm, in fact, does not provide better results in terms of loss reduction. However, it achieves similar loss values and flatter voltage profile at a lower cost.

The NSGA-II algorithm is also applied to the scenario with feeder capacitors already installed. The Pareto front for both the NSGA-II and modified Grainger/Lee methods for CASE 2 is shown in Figure 4. The modified Grainger/Lee solution is undoubtedly inferior on all accounts compared to the solutions found using the genetic algorithm. As already mentioned, the Grainger/Lee method is not optimal for cases when capacitors are already present.

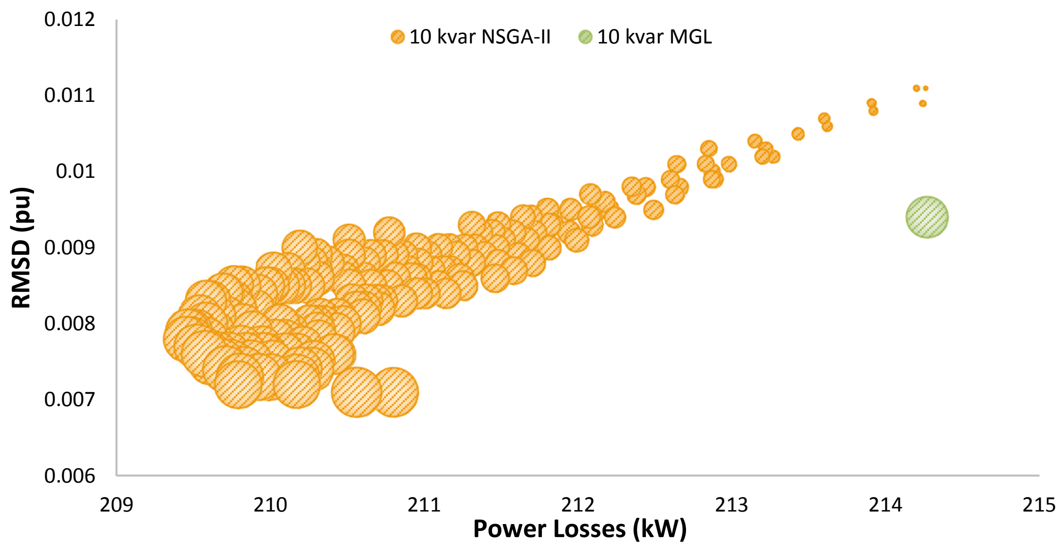

Figure 4: Pareto front of the solutions obtained with the NSGA-II and the modified Grainger/Lee (MGL) method, applied to EPRI Circuit 5 with feeder capacitors previously installed. The larger the bubble is the higher the cost. The larger the bubble is the higher the cost ($0 – 106k).

Another interesting observation is that minimizing the power losses and the RMSD objectives start to show their contrasting behavior for solutions with larger numbers of installed capacitors. The increasing demands, due to the increased voltages with the existing capacitors, results in the hockey stick shape in the Pareto front seen in Figure 4.

CONCLUSION

In conclusion, the modified Grainger/Lee methodology has been shown to be applicable for any size capacitor at any voltage level. Using multiple small capacitors can yield a greater reduction in power losses and provide a flatter voltage profile – compared to the traditional larger capacitors banks. Utilizing smaller capacitors is also more expensive ($/var) compared to larger ones, but can be useful to solve local low voltage, power factor, or capacity issues. However, the traditional method can provide a good, but not optimal solution when capacitors are already present in the feeder.

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