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### Hosting Capacity Optimization Using Linearized AC Power Flow Analysis

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#### SUMMARY

The environmental, economic, and performance benefits gained by incorporating distributed generation (DG) into electric power systems are driving interest in the accommodation of DG. However, accommodation of DG into distribution systems has created challenges for distribution system operators. Many distribution systems are designed in a radial structure convenient for transferring electric power from a central location to consumers at the peripheries, i.e., a one-way flow of electricity. A shift towards DG has raised two concerns: (1) how will introducing DG into existing transmission and distribution systems affect their operational performance and (2) what is the best way to incorporate DG into existing distribution systems without jeopardizing system performance? These questions highlight the need to understand the impact of DG on an active network. A straightforward method for understanding the effect of DG on power system performance is the hosting capacity approach. Hosting capacity is defined as the amount of new production, or consumption, that can be connected to the grid without adversely impacting the reliability or voltage quality for other users. The study of hosting capacity is commonly accomplished by simulating power flow for each potential placement of DG while enforcing operating limits (e.g. voltage limits and line thermal limits). Traditionally, power flow is simulated by solving full nonlinear AC power flow equations for each potential configuration. This is computationally intensive due to the nonlinearity of the equations and the requirement of a large number of iterations to check all possible DG capacities and locations. In this paper, an efficient optimization method is developed, which linearizes the power flow equations and leads to a dramatic reduction in the optimization problem's complexity. This increases the speed and robustness of the hosting capacity method, allowing for real-time analysis of a radial distribution system. The effectiveness of the method is showcased on the IEEE 33-bus distribution test system and its results are compared to a traditional nonlinear hosting capacity method.

#### KEYWORDS

Distributed generation, Hosting capacity, Linear power flow analysis

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## 1. INTRODUCTION

Distributed generation (DG) units such as solar, wind, or geothermal power are playing an increasingly important role in power systems in terms of loss reduction, voltage quality improvement, and decreasing environmental impacts. At the same time, this growing integration has created some unique challenges for distribution systems. Network infrastructure was intended to transmit electric power produced from large centralized power plants (e.g. coal, hydrothermal, nuclear) miles away to the end consumers. With this goal in mind, most distribution grids were designed in a radial structure with line carrying capacity being higher for lines closer to the generation. Thus, these grids were not intended to carry high levels of energy produced by DG near consumer areas. However, current infrastructure of distribution systems can accommodate some DG units. The amount of DG power that a network can host before adversely affecting performance is known as the hosting capacity. Determining the hosting capacity requires demonstrating that a distribution system operating at the hosting capacity limit does not exceed its operating specifications. Proper operation of a distribution system requires that bus voltages remain within imposed voltage ranges and that power flow through distribution lines remain within acceptable thermal limits. To demonstrate that an acceptable operation is achieved, a distribution system is simulated with the desired configuration and its performance indices (e.g. voltages, active and reactive power flows) are compared with associated limits [1].

Several hosting capacity optimization methods have been proposed in the literature able to contribute to increase the penetrations of DG while enforcing voltage and thermal limits [2]-[4], power loss minimization [5]-[6], and voltage regulation process [7]. These methods calculate the power flows at all points in the system using AC power flow analysis. However, while the AC power flow equations are straightforward to formulate they will show a drawback when integrated within the hosting capacity method. Typical computational strategies include an iterative approach to optimize distribution hosting capacity using these equations. The need for optimization-via-iteration follows from requiring a solution to the AC power flow equations for each potential system topology. Iteration requires high computation time when investigating large search spaces. This limitation prevents accurate optimization since accuracy requires analyzing many potential system solutions. Each of these system solutions requires an iterative solution to the AC power flow equations, compounding the computational burden.

Traditional methods thus can potentially arrive at the crossroads of speed and accuracy. A large increment analysis can yield a quick optimization, but with weak confidence in the result [8]. However, a small increment between iterations will result in a reliable optimization, but at the cost of much computational power [9]. This paper addresses the abovementioned issues by linearization of the nonlinear AC power flow equations. With these equations, linear optimization can be performed instead of an iterative optimization. This allows a fast scanning of a huge search space, exact solutions, and rapid computation. Additionally, linear analysis does not require iterations, eliminating concerns about proper convergence. A potential drawback, however, is the error introduced by the deviation of the linearized model, which can be shown as negligible.

The balance of the paper is organized as follows: Section 2 presents the proposed model including the assumptions used to linearize the power flow equations. Section 3 presents hosting capacity simulation results on the IEEE 33-bus distribution test system and compares them with the nonlinear simulation results. Section 4 concludes this paper.

## 2. HOSTING CAPACITY MODEL

Hosting capacity is the amount of new production, or consumption, that can be connected to the grid without adversely impacting the reliability or voltage quality for other users. The hosting capacity optimization is a method that determines the operating state in which the hosting capacity is maximized without degrading the system performance. To fully define the problem, the meaning of “degrading performance” and “operating states” should be clarified. Here, a system’s performance is considered degraded if its bus voltage magnitudes exceed associated limits, if active and/or reactive power flow through a bus exceeds component limits, or if active and/or reactive power flow through lines exceed the line capacity limits. The “operating states” of the system are considered to be possible DG injection profiles in the network. This definition of operating states is chosen to investigate hosting capacity improvements without changing the existing network infrastructure. Network infrastructure is assumed to be radially connected since this is the most common structure in distribution networks. Other working definitions for “degrading performance” and “operating states” may be defined, leading to different hosting capacities that emphasize different considerations.

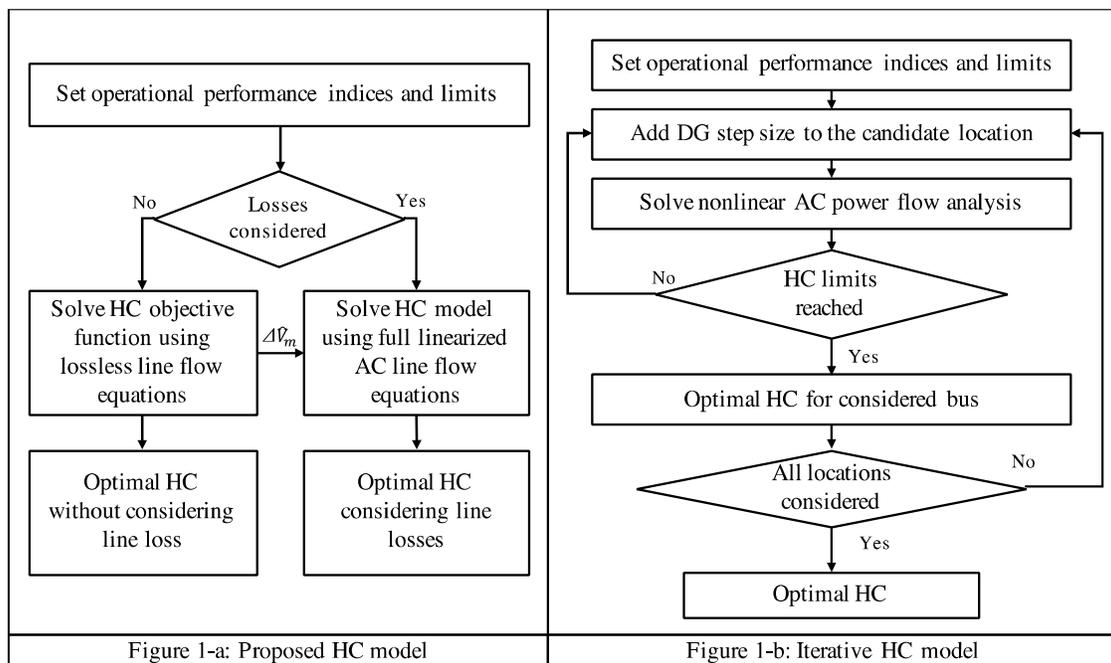


Figure 1: A Comparison between proposed HC approach and traditional iterative HC model.

The objective of the method outlined here is to streamline the hosting capacity optimization computation. This is accomplished by linearizing the AC power flow equations and dramatically reducing the steps required. Figure 1 compares the hosting capacity optimization procedure for the proposed linear method (Figure 1-a) and for the traditional nonlinear method (Figure 1-b). The difference in number of required steps arises from the iterative loops included in the nonlinear algorithm. This figure shows two iterative loops. The first iteration requires a nonlinear AC power flow solution for each increment of injected DG until performance limits are reached. The second iteration requires that the first iteration be performed for each potential DG location. There is also a third iteration hidden in the “solve nonlinear AC power flow analysis” step, as solving these equations employs an iterative approach. All in all, three nested iterations are embedded in the traditional approach to the problem. In contrast, the linear method uses a serial algorithm – requiring only one iteration when solving the full AC power flow equations.

The serialization of the algorithm was accomplished by creating a linearized power flow model. The model expands upon traditional AC power flow equations. The objective function is to maximize the hosting capacity (i.e. the maximum installed amount of DG). To simulate network behavior, the objective function is subject to the active and reactive power balance equations and network power flow. These equations ensure that the power-in plus power generated equals the power-out plus power consumed in each bus. The nonlinear active and reactive power flow through line connecting buses  $m$  and  $n$  are defined as follows:

$$PL_{mn} = g_{mn} V_m^2 - g_{mn} V_m V_n \cos(\theta_m - \theta_n) - b_{mn} V_m V_n \sin(\theta_m - \theta_n) \quad \forall mn \in L \quad (1)$$

$$QL_{mn} = -b_{mn} V_m^2 - b_{mn} V_m V_n \cos(\theta_m - \theta_n) - g_{mn} V_m V_n \sin(\theta_m - \theta_n) \quad \forall mn \in L \quad (2)$$

Here  $g_{mn}$  and  $b_{mn}$  are respectively the conductance and the susceptance of the line connecting buses  $m$  and  $n$ . Variables  $V$  and  $\theta$  are respectively the voltage magnitudes and voltage angles in each bus. To linearize the power flow equations, two assumptions will be made about the radial network. The first assumption is that the multiplication of bus voltage deviation from the Point of Interconnection (POI) and the change in phasor angles, i.e.  $(\Delta V_m - \Delta V_n) (\Delta \theta_m - \Delta \theta_n)$ , is small for all buses. The POI is the point at which the distribution network connects with the upstream grid. The  $\Delta V_m$  may be written as  $V_m = V_{POI} + \Delta V_m$ , where  $V_m$  is the voltage phasor at bus  $m$  and  $V_{POI}$  is the fixed voltage phasor at the POI. To simplify the analysis, the voltage phasor units will be in per units such that  $V_{POI} = 1 \angle 0^\circ$  p.u. Here  $\Delta V_m$  is the normalized difference between the voltage at bus  $m$  and the voltage at the POI, which is usually kept within  $\pm 0.1$  p.u. Similarly, the phasor angles of all buses are redefined based on the POI voltage phasor angle such that  $\theta_m = \theta_{POI} + \Delta \theta_m$ . The second assumption is that between adjacent buses  $m$  and  $n$  the voltage phase difference,  $\Delta \theta_m - \Delta \theta_n$ , is close enough to zero to allow a small angle approximation of trigonometric functions. Applying the approximations and neglecting the products of small terms, the AC power flow equations can be expressed as follows:

$$PL_{mn} = g_{mn} (\Delta V_m - \Delta V_n) - b_{mn} (\Delta \theta_m - \Delta \theta_n) + g_{mn} \Delta V_m (\Delta V_m - \Delta V_n) \quad \forall mn \in L \quad (3)$$

$$QL_{mn} = -b_{mn} (\Delta V_m - \Delta V_n) - g_{mn} (\Delta \theta_m - \Delta \theta_n) - b_{mn} \Delta V_m (\Delta V_m - \Delta V_n) \quad \forall mn \in L \quad (4)$$

The first two terms are linear, but the last term is still nonlinear. By ignoring losses, the nonlinear terms can be removed, and a set of linearized equations are derived. Solving the lossless equations yields a set of voltage values,  $\Delta V_m$ . By substituting the  $\Delta V_m$  factor in the third term in (3) and (4) with the calculated values from the lossless power flow equations, full linearized AC power flow equations will be derived. A key difference between the lossless and the full linearized equations is that the lossless equations are forced to obey the lossless conditions,  $PL_{mn} + PL_{nm} = 0$  and  $QL_{mn} + QL_{nm} = 0$ .

These linear power flow equations are solved within the hosting capacity optimization method shown in Figure 1a. With this formulation, the major technical challenge for implementing the proposed method has been overcome. To simulate performance constraints, the active and reactive power flow limits, as well as voltage limits, are incorporated into the model. The considered line flow constraints place limits on the maximum and minimum amount of power that may flow through the lines. Additionally, the voltage magnitude values are enforced by the upper and lower voltage magnitudes limits in all buses.

### 3. SIMULATION RESULTS

The effectiveness of the proposed linearized model is showcased on the IEEE 33-bus test system shown in Figure 2. The system contains 33 buses and 32 lines. The results, including runtime, of the linearized algorithm are compared with the results of the iterative nonlinear hosting capacity algorithm. Both algorithms are initialized with the same parameters (i.e. nodal loads, line flow limits, and voltage limits) to enable a direct comparison.

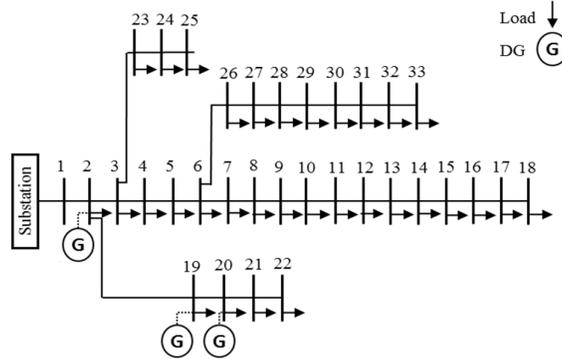


Figure 2: The IEEE 33-bus distribution system.

The traditional hosting capacity optimization approach is restricted by computational requirements. To demonstrate one manifestation of this restriction, the resolution of the hosting capacity was increased and the runtime was measured. Hosting capacity resolution can be increased in the iterative approach by reducing the DG step size during each iteration. With reduced step size, more values of DG injection power are sampled in a given range at the cost of requiring more iterations. Four DG step sizes were chosen for this demonstration, 1 kW, 10 kW, 100 kW, and 1 MW. To avoid impractical computation times, DG generation was only swept in one location at a time. Figure 3 shows the relationship between accuracy and time. A trade-off emerges in which decreasing error causes an increase in computation time and decreasing computation time causes an increase in error. For DG step sizes of 1 kW, 10 kW, 100 kW and 1 MW the computation time is 472 s, 49 s, 6 s, and 2 s, respectively.

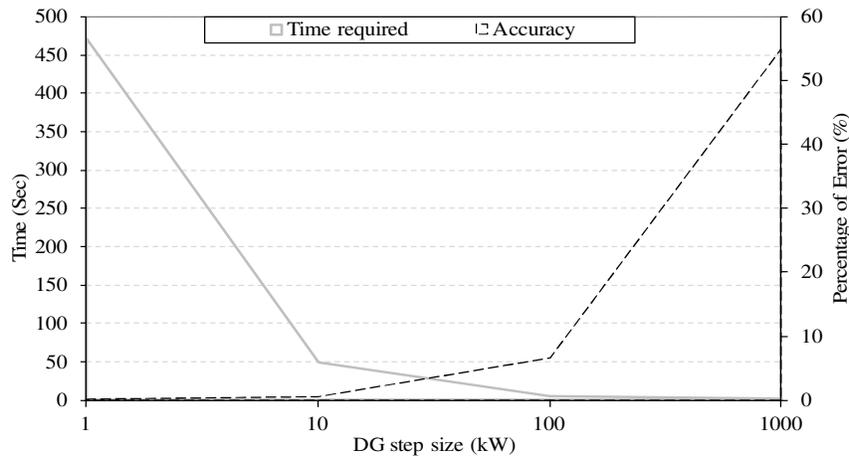


Figure 3: Tradeoff between speed and accuracy for each DG step size based on the iterative method.

While it is feasible to find a balance between accuracy and time in this case, it becomes infeasible to do so when trying to expand the analysis, e.g., to optimize DG placement to multiple buses simultaneously. The linearized hosting capacity optimization was performed on

the same system and results are compared to the highest fidelity iterative optimization executed (DG step size of 1 kW). Table 1 summarizes the results of the two methods with the total DG hosting capacity of each individual bus differing by, on average, 0.32% of error. The total optimized hosting capacity was also compared between the two methods. The optimal result here is a total hosting capacity of 8.518 MW for the traditional method and 8.484 MW for the proposed method, a difference of 0.41%.

Table 1: A comparison between the proposed method and the traditional iterative method.

	<b>Only one bus at a time is considered</b>		<b>All buses are considered</b>	
	<b>Time required</b>	<b>% of Error</b>	<b>Time required</b>	<b>% of Error</b>
<b>Proposed model</b>	1.2 sec	0.32	4 sec	0.41
<b>Iterative method</b>	472 sec	-	1032 hours	-

Comparing runtimes, the linearized optimization method vastly outperforms the traditional iterative method. When analyzing the hosting capacity of each bus independently, average run times for the traditional iterative method is approximately 472 seconds, while the linearized method averages 1.2 seconds. When trying to optimize hosting capacity for all buses simultaneously, the iterative method has a total runtime of 1,032 hours. This is because the iterative method must run through different permutations of DG injections at all bus locations to find the optimal hosting capacity. The linearized method reproduces this result in 4 seconds. As a last check, the accuracy in obtaining voltage magnitudes is compared, showing a total difference of less than 0.07% in any of the buses between two methods. This result advocates that the linearized hosting capacity optimization method is able to reproduce the results of the iterative method nearly identically. It further highlight that the linearized method does not need to consider the same speed-accuracy tradeoff as in the traditional iterative method.

#### 4. CONCLUSION

Hosting capacity optimization is a critical study used to determine the maximum DG capacity able to be injected into a distribution system without negatively impacting its operational performance. A linearized method for hosting capacity optimization was presented and compared to the traditional iterative method of hosting capacity optimization. Traditional methods are prized for their high accuracy as they are based on full AC power flow equations. Nonlinearity of these equations, however, has bottlenecked the applications of the method. This bottleneck appears as a speed vs accuracy tradeoff. The tradeoff was demonstrated by implementing the traditional hosting capacity optimization method with different DG step sizes. As the step size increased, computation time decreased and error increased. The linearized method presented here has been demonstrated to agree nearly identically with the traditional iterative method. The overall hosting capacity showed an agreement within 0.41% error, and the bus voltage magnitudes agree within less than 0.07% of error. These capabilities would be ideal for use in network management operations to optimize the accommodation of DG within existing radial network infrastructure.

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