### Trajectory Sensitivity Analysis as a Means of Performing Dynamic Load Sensitivity Studies in Power System Planning

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### Load modeling for power system planning

- Load modelling is one of the most important aspects of time-domain simulations for power system planning.
- North American Reliability Corporation (NERC) requires system peak load levels to be represented by load models considering behavior of induction motors <sup>1</sup>
- Significant work done by the WECC load modelling task force led to the development of the composite load model (cmpldw).
- As the NERC standards become enforced in the near future, utilities in North America will need to adopt dynamic load model structures similar to the WECC composite load model (*cmpldw*).

<sup>1.</sup> NERC TPL Standard -001-4 "System peak load levels shall include a load model which represents the expected dynamic behavior of loads that could impact the study area, considering the behavior of induction motor loads."

### Challenges in load modeling

- The WECC composite load model represents an aggregation of different types of load at the substation node.
- The exact composition of different types of loads in the composite load model is not known during the planning stage.
- This introduces significant uncertainties into the model.
- To deal with uncertainties, it becomes important to develop a systematic approach for performing load sensitivity studies.
- This paper presents some initial research results for using a methodology known as trajectory sensitivity analysis for performing load sensitivity studies.

### Trajectory sensitivity analysis

A power system can be represented by a set of differential algebraic equations

$$\dot{x} = f(x, y, \lambda) \tag{1}$$

$$g^{-}(x, y, \lambda) = 0 \text{ for } y_{k} < 0 \tag{2}$$

$$g^{+}(x, y, \lambda) = 0 \text{ for } y_{\nu} \ge 0 \tag{3}$$

- $\star$  x represents network dynamic states (rotor angles, frequency, flux)
- $\diamond$  y represents network algebraic variables (network voltages and angles)
- $\star$   $\lambda$  represents the systems parameters of interest (for our case  $\lambda$  represents the load parameters)
- Equations (2) and (3) represent the pre and post network algebraic equations following a discrete event. For example, it could represent the power consumed by a  $1\phi$  induction motor (modeled by algebraic eqs e.g ld1pac) before and after it stalls. The stalling conditions are modeled in the form  $y_k=0$

### Trajectory sensitivity analysis

- Sensitivity of state variables to parameter  $\lambda$  is given by  $\frac{\partial x(t)}{\partial \lambda}$
- $\Leftrightarrow$  Sensitivity of algebraic variables to parameter  $\lambda$  is given by  $\frac{\partial y(t)}{\partial \lambda}$
- The predicted trajectories can be the calculated by a first order approximation as

$$x(t)_{pred} = x(t)_{old} + \frac{\partial x}{\partial \lambda} \Delta \lambda$$
 (4)

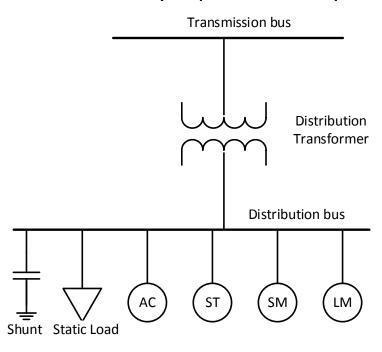
$$y(t)_{pred} = y(t)_{old} + \frac{\partial y}{\partial \lambda} \Delta \lambda$$
 (5)

### Trajectory sensitivity analysis

- Calculation of the sensitivities requires the power flow Jacobian at each step of the time domain solution.
- The power flow Jacobian is available as a by-product of a traditional time domain simulation routine, which incorporates an implicit integration algorithm (trapezoidal rule is an example of an implicit integration algorithm)
- Runtime availability of the power flow Jacobian reduces the computation effort in evaluating the sensitivities and it can be done simultaneously while performing a time domain simulation
- The sensitivities of different parameters are independent of each other. These can be evaluated in parallel using an appropriate parallel computing architecture, enabling additional savings in time

### System and load model

- The WECC 2012 peak summer case was used in the study. The study was performed using a Matlab based open source software package PSAT
- ❖ A WECC composite load model was used in order to ensure that the test case closely represented a practical real-life scenario.



Composite load model

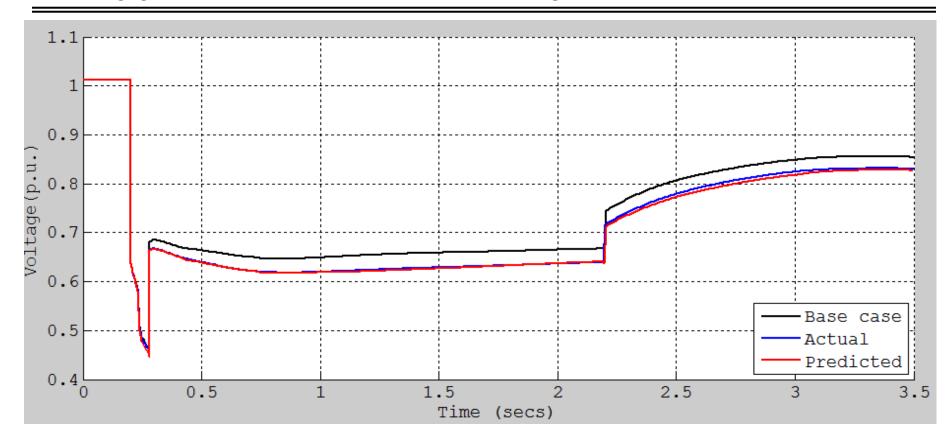
WECC 2012 Case		
Buses	17047	
Generators	2 059	
Transformers	5 727	
Branches	13 178	
Loads	6 781	
Loads represented by composite load model	3729	

LM	Large motors
SM	Small motors
ST	Trip motors
AC	Air conditioners

### Application to the WECC system

- Trajectory sensitivity analysis was used to study the effect of change in load composition at different buses on the system algebraic and state variables following a single disturbance.
- ❖ A WECC 2012 summer peak system model was used for this work. A threephase fault was applied on a major 500 kV line. The fault is cleared by opening the line after 5 cycles.
- The sensitivity of voltage and frequency to percentage changes in the airconditioning (AC) load at 20 buses were studied.
- ACs were observed to stall at these 20 buses and hence these 20 buses were selected to study the effect of change in load composition.
- $\Delta K_p$  refers to the percentage change in AC load at each bus

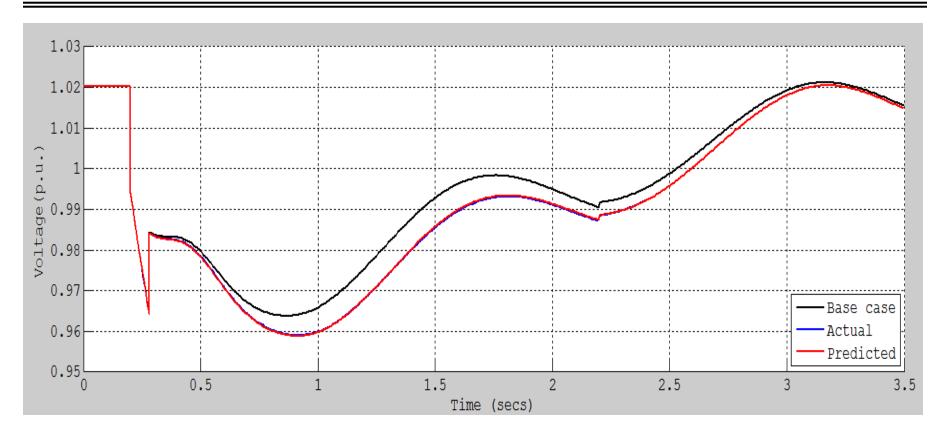
### Application to the WECC system



Actual and predicted voltage at bus 611 for a  $\Delta$ Kp of 5 at 20 buses

This load bus is electrically close to the fault location

### Application to the WECC system



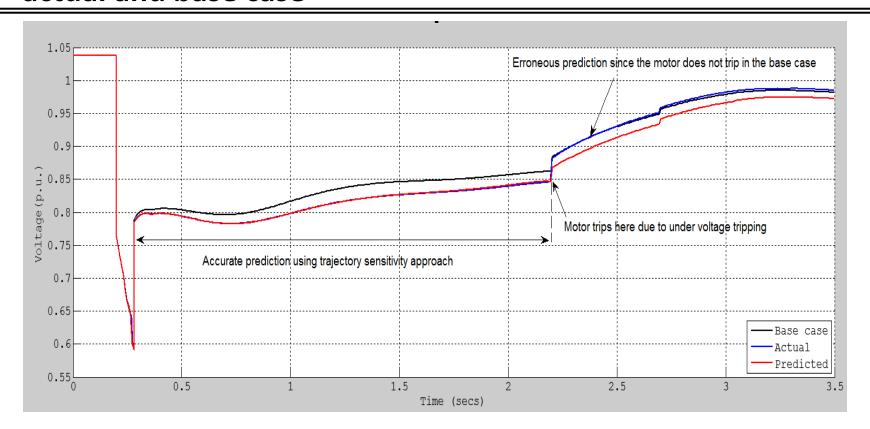
Actual and predicted voltage at bus 212 for a  $\Delta$ Kp of 5 at 20 buses

This load bus is electrically far from the fault location

# Effect of relay, contactors and discontinuous load characteristics

- Relays and/or contactors and discontinuities in the load characteristics introduce severe non linearity in the load models.
- Non linear models lead to approximation errors in a trajectory sensitivity based approach.
- Trajectory sensitivity based approach can be erroneous when the base case and the actual case to be predicted do not encounter and traverse same switching surfaces.
- ❖ For example, the base case and actual scenario (to be predicted) should have same number of motors tripping (motorw model) and same number of air-conditioners (stalling / restarting) to get an accurate linear prediction of trajectories.
- For the present scenario simulated, air-conditioners account for majority of the load. Stalling of additional AC units has a pronounced effect on the linear approximation.

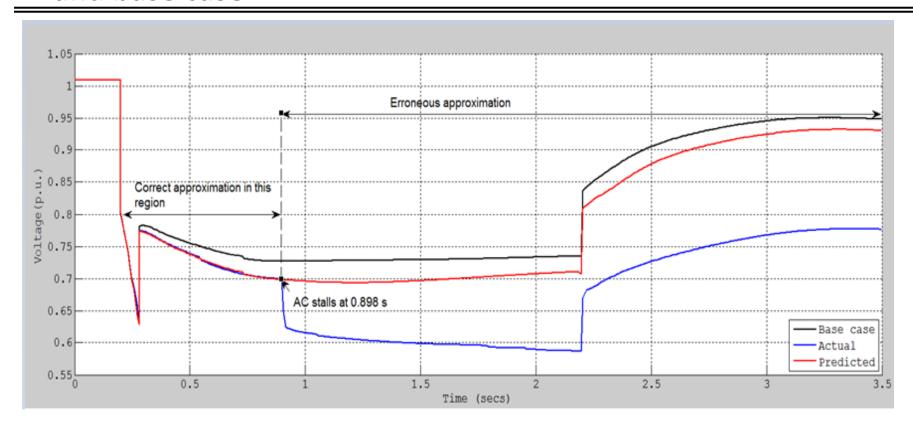
## Effect of different number of small motor (motorw) tripping in actual and base case



Actual and predicted voltage at bus 419 for 5 percent load change at 20 buses

- ❖ 14% of the total load is represented by small motor which is relatively small.
- Effect on error in trajectory approximation is localized

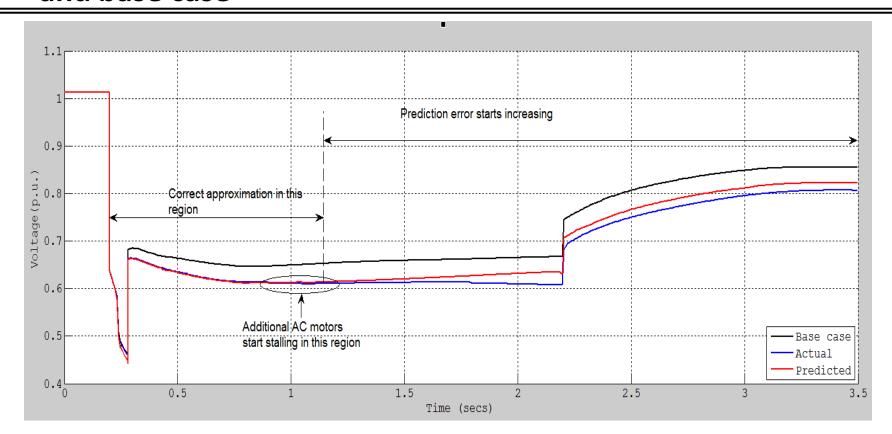
## Effect of different number of air conditioners stalling in actual and base case



Actual and predicted voltage at bus 420 for 6 percent load change at 20 buses

- ❖ 30% of the total load is represented by air-conditioners.
- The effect is more pronounced at this bus than at buses further away.

## Effect of different number of air conditioners stalling in actual and base case



Actual and predicted voltage at bus 611 for 6 percent load change at 20 buses

The effect of different number of air-conditioners stalling can be seen but the error is less pronounced.

Routines	Time (N-R without optimal multiplier)	
Time domain simulation (includes storing Jacobian)	3487.729 sec	
Calculate initial values of sensitivities	1.708 sec	
Calculate the sensitivities	913.87 sec	
Create the final trajectory	32.29 sec	
Total time	4435.59 sec (74 mins)	
Size of file containing Jacobian entries	9 GB	

❖ It should be noted that a trapezoidal method of integration is used in the trajectory sensitivity analysis. The increased simulation time is due to increased Newton-Raphson iterations required for convergence of solution during disturbances

- ❖ To reduce the simulation time an optimal multiplier is introduced
- The intermediate step of a time domain simulation can be stated as a set of nonlinear equations given by

$$f(x) = 0 (6)$$

 $\clubsuit$  If the initial guess of the solution vector is  $x_0$  then the updated solution vector can be calculated as

$$-f(x_0) = J\Delta x \tag{7}$$

$$x_1 = x_0 + \Delta x \tag{8}$$

J: Jacobian matrix containing the partial derivatives of f w.r.t x

 $\Delta x$ : Correction vector by which  $x_0$  is incremented

 $\diamond$  NR method can be modified by introducing an optimal multiplier  $\alpha$ , such the new estimate for the solution is given by

$$x_1 = x_0 + \alpha \Delta x \tag{9}$$

lacktriangle The value of lpha is calculated by solving a one dimensional minimization problem

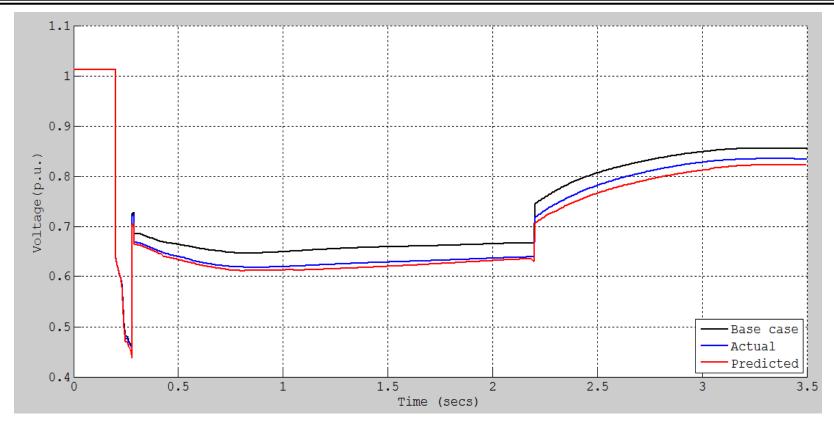
$$\alpha = \arg\min(f(x_0 + \alpha \Delta x)^t f(x_0 + \alpha \Delta x)) \tag{10}$$

- The exact solution of the one dimensional optimization problem in (10) is time consuming and a local minimizer can be obtained by methods like line search algorithms or interpolation methods.
- ❖ A cubic interpolation method has been used for this study.

Routines	Time (N-R without optimal multiplier)	Time (N-R with optimal multiplier)
Time domain simulation (includes storing Jacobian)	3487.729 sec	1695.548 sec
Calculate initial values of sensitivities	1.708 sec	No change
Calculate the sensitivities	913.87 sec	353.97 sec
Create the final trajectory	32.29 sec	No change
Total time	4435.59 sec (74 mins)	2083.108 sec (35 mins)
Size of file containing Jacobian entries	9 GB	5 GB

A comparison of the two approaches shows that using an optimal multiplier results in a substantial reduction in computation time

## Effect of different number of air conditioners stalling in actual and base case



Actual and predicted voltage at bus 15611 for 5 percent load change at 20 buses (N-R method with optimal multiplier)

Using an optimal multiplier does not introduce any significant error in trajectory approximation.

#### Conclusions

- One of the key challenges in load modeling is determining the composition of aggregated model parameters.
- An approach to address this issue is the application of load model sensitivity analysis using trajectory sensitivity.
- The main benefit of this method is that it allows a planner to study multiple scenarios with uncertain load parameters without the need of multiple simulations.
- Multiple sensitivities can be computed in parallel enabling additional savings in time.
- The main disadvantage at present is that being a linear approach it cannot sufficiently handle severe non-linearity in load models.
- The computation time is relatively higher due to the use of an implicit integration algorithm. However it can be reduced substantially by introducing optimal multipliers.