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Viewing the Load Flow Equation via Power, Voltage, and Discriminant Triangles

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SUMMARY

We describe the development and application of three power and voltage triangles that enable the power load flow equation parameters to be viewed on the triangles.

1. The Load Flow Power Triangle contains the scalar quantities of the load flow equation on the triangle sides while the triangle angles capture the load flow equation angle information.
2. Next, we apply a suitable scaling multiplier to the Load Flow Power Triangle sides creating the Load Flow Voltage Triangle. The Load Flow Voltage Triangle shows the relationship of the V_u and V_l voltages on the PV curve at an operating point.
3. Finally, we describe a Load Flow Voltage Discriminant Triangle as a hybrid of the other two triangles. The Load Flow Voltage Discriminant Triangle is unique because it provides a solution to the load flow equation and indicates the closeness of the solution to the critical bifurcation point.

We note throughout the paper, the results of the triangles are consistent with the findings in the literature in [1,2,4,5,7,9].

We present examples illustrating use of the Load Flow Voltage Triangle to identify buses with the worst V_l /V_u voltage ratios from a large power system in Example 2. In addition, we note voltage monitoring situational awareness opportunities for the system operator.

KEYWORDS
Load flow, triangle, voltage discriminant

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Introduction

In the following, a variable followed by an angle symbol denotes a complex phasor quantity ($\rho\angle\beta$) while a variable without an angle symbol (ρ) denotes the magnitude of the phasor quantity.

In this section we lay the groundwork to express the load flow equation in polar form. The $\rho\angle\beta$ polar component combines the series impedance element with the associated power flow delivered by the element. The $\rho\angle\beta$ polar form allows equations to be viewed in a concise graphical triangle format and provides us with intuitive insights.

Using the system in Figure 1, we define $\rho\angle\beta$ equal to the complex product $[(P+jQ)^* (R+jX)]$ where the subscripted * denotes the complex conjugate. The load flow equation can be written as

$$V\angle\phi = E\angle 0 - \rho\angle\beta / V\angle -\phi \qquad (1)$$

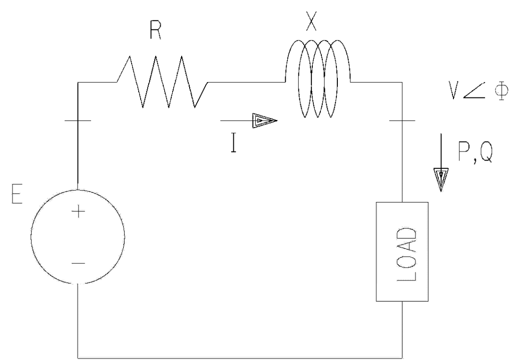

Figure 1 – Simple circuit for load flow equation

Separating (1) into its real and imaginary parts we get:

$$EV\cos(\phi) = V^2 + \rho*\cos(\beta) \qquad (2)$$
$$-EV\sin(\phi) = \rho*\sin(\beta) \qquad (3)$$

We eliminate the phase angle ϕ from (2) and (3) by applying solution techniques similar to those used in references [1] and [2].

The resulting fourth degree polynomial expressed in polar form is:

$$0 = V^4 + [2\rho*\cos(\beta) - E^2]V^2 + \rho^2 \qquad (4)$$

The fourth degree polynomial makes V a function of E, ρ, and β.

Contraint (5) needs to be true for a valid solution.

$E^2 > 2\rho[1+\cos(\beta)]$ - Two unique pairs of real roots (5)
$E^2 = 2\rho[1+\cos(\beta)]$ - Pair of identical roots (6)
$E^2 < 2\rho[1+\cos(\beta)]$ - Complex roots (7)

This approach does not support intuitive insights to view V as a function of E, ρ, and β. The remainder of this paper identifies a more practical approach by employing the Load Flow Power, Load Flow Voltage, and Load Flow Voltage Discriminant Triangles in the next section.

Triangle Descriptions

Load Flow Power Triangle

The Load Flow Power Triangle in Figure 2 encapsulates the load flow equation (1) parameters E, V, and ρ as scalars. You will observe that V^2, EV, and ρ lay on the triangle sides. Additionally, the triangle angles contain information on (1) ϕ and β parameters as scalars.

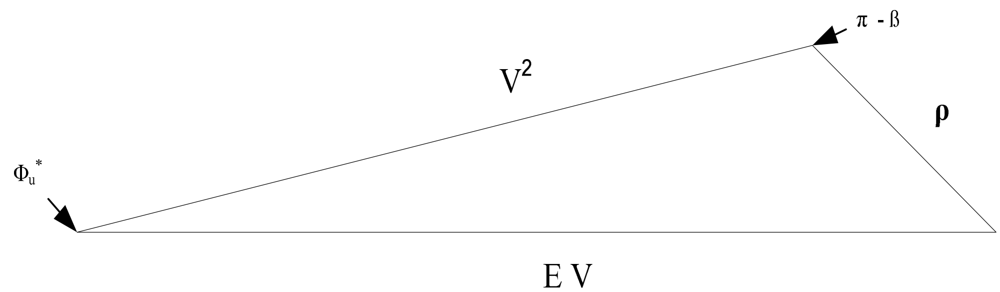

Figure 2 – Load Flow Power Triangle

Applying equation (6) to (4) we find V^2 equals ρ at the critical bifurcation point. Consequently, the Load Flow Power Triangle becomes an isosceles triangle for this scenario. We use an isosceles triangle to solve simple lagging, unity, and leading power factor scenarios in Example 1 below.

Load Flow Voltage Triangle

We note that (4) has two distinct positive real voltage solutions for constraint (5). In the context of a PV curve, let V_u symbolize the stable upper solution while V_l denotes the unstable lower solution. Making use of Vieta's formulas in reference [3] on (4), it can be shown the product of the voltage solution magnitudes V_u times V_l equals ρ. In a more general way (8) below incorporates the phase angle information subject to the constraint the voltage phase angles are referenced to the voltage E∠0.

$V_u^* \, V_l^* = \rho\angle\beta$ where* represents the complex conjugate of the voltages $V_u\angle\phi_u$ and $V_l\angle\phi_l$. (8)

Applying (8) to the Load Flow Power Triangle, we constructed the Load Flow Voltage Triangle in Figure 3.

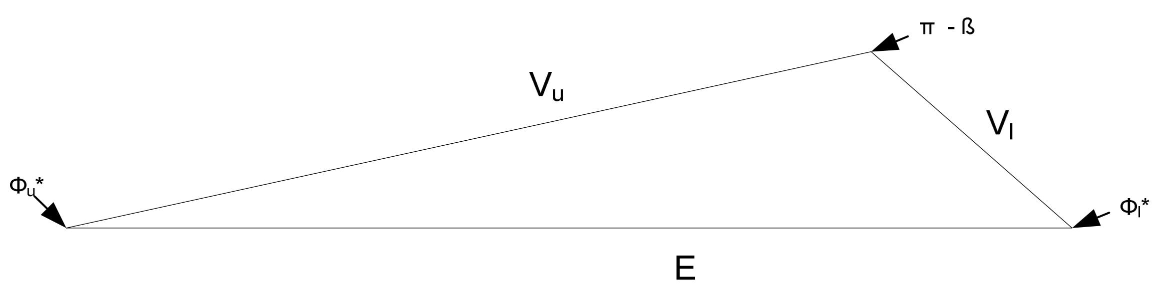

Figure 3 – Load Flow Voltage Triangle

The Load Flow Voltage Triangle shows the relationship of E, V_u, and V_l voltage magnitudes and the associated phase angles ϕ_u, ϕ_l, and β. Observe that the angle between E and V_u is ϕ_u^*

where * denotes the conjugate value. At the apex of the standard PV curve, we know V_u equals V_l. Therefore, we define the Triangle Voltage Stability Index (TVSI) as the ratio of V_l/V_u voltage magnitudes. When TVSI equals 1, then we are at the critical bifurcation point.

In this discussion, we assume the impedance Z equals the |R+jX| magnitude. From (1) and (8) it can be shown the voltage V_z equals V_l where V_z is the voltage drop across the series impedance Z. Consequently, TVSI is similar to defining the TVSI as the ratio of V_z/V_u magnitudes. This is consistent with reference [4] which notes the voltage drop across the transmission impedance equals the load voltage at a maximum power condition.

We demonstrate the practical application of the triangle in Example 2 below.

In [8] the voltage stability indices use impedance and voltage data to calculate the indices. In our approach, we calculate the indices from the V_l/V_u voltage magnitudes ratios.

Load Flow Voltage Discriminant Triangle

The Load Flow Voltage Discriminant Triangle in Figure 5 is a composite of the Load Flow Power and Load Flow Voltage triangles. In developing this triangle, we discovered the real component of the receiving voltage V_u equals E/2 + Δ, implying Δ could be used as a measure of voltage stability. When Δ equals zero, then V_u equals V_l.

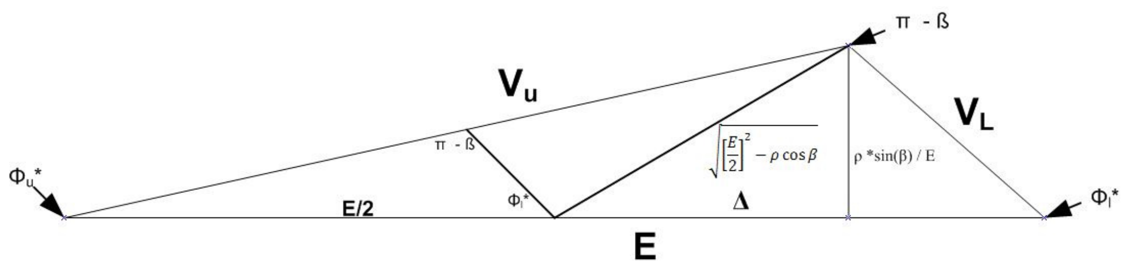

Figure 5 – Load Flow Voltage Discriminant Triangle

By knowing E, V_u, and ϕ_u from a solved load flow study or state estimator run, we can readily calculate Δ using equation (9). We utilize (9) in the analysis for Example 2.

$$\Delta = V_u{*}\cos(\phi_u) - E/2 \qquad (9)$$

When Δ equals zero in (9), the loci of critical voltage is given by (10).

$$[Vu/E]\cos(\phi_u) = ½ \qquad (10)$$

Equation (10) agrees with reference [5] findings.

We also note from Figure 5, the real component of $V_u\angle\phi_u$ equals E/2 when Δ equals zero.

This triangle can provide the solution to the load flow equation by calculating Δ from the interior triangle sides. From Figure 5 we can deduce Δ^2.

$$\Delta^2 = (E/2)^2 - \rho{*}\cos(\beta) - [\rho{*}\sin(\beta)/E]^2 \qquad (11)$$

The complex voltage solutions $V_u\angle\phi_u$ and $V_l\angle\phi_l$ are then given by:

$V_u \angle \phi_u = (E/2 + \Delta, -\rho * \sin(\beta) / E)$, and (12)

$V_l \angle \phi_l = (E/2 - \Delta, -p * \sin(\beta) / E)$. (13)

Equations (12) and (13) are consistent with reference [9] findings.

The Voltage Discriminant Triangle is unique in providing a solution to the load flow equation along with an indication of the closeness of the solution to the critical bifurcation point. The size of the Voltage Discriminant Δ is used to identify potential voltage problems in Example 2. We outline the derivation of (11) below.

1. Draw the median cevian line segment from the π-β vertex to the E/2 midpoint.
    a. Use Stewart's Theorem [6] on the median to calculate the cevian line segment length.
        i. Cevian length^2 = $[2V_u^2 + 2V_l^2 - E^2] / 4$
        ii. Apply Vieta's sum of roots formula [3] and equation (4)
            1. $V_u^2 + V_l^2 = E^2 - 2\rho\cos(\beta)$
    b. After simplifying, we arrived at the median cevian length noted in Figure 5.
2. Draw a cevian line segment from the π-β vertex perpendicular to E.
    a. Apply Law of Sines
        i. $\sin(\phi_u)/V_l = -\sin(\beta)/E$
        ii. Replace $\sin(\phi_u)$ with $\mathrm{Im}(V_u) / V_u$
        iii. Multiply 2ai by $V_u * V_l$ magnitudes
    b. Using V_u and V_l magnitudes, apply (9) and simplify.
    c. After simplifying we arrive at the cevian length noted in Figure 5
        i. $\rho\sin(\beta)/E$
3. Equation (11) follows from 1b and 2ci.

Examples

Example 1: Lagging, Unity, and Leading Power Factor

In this example, we are using the circuit depicted in Figure 1.

This example demonstrates the application of the triangles to three simple voltage stability problems at the critical bifurcation point. At the critical point, we know from Figure 3 that V_u equals V_l magnitude. Consequently, we utilized standard isosceles triangle math in this example.

Calculate the per unit receiving V_u voltage, phase angle ϕ_u, and real power using the Load Flow Voltage Triangle and equation (8) for the 0.9 lagging, unity, and 0.9 leading power factors.

Assumptions:

E = 1 PU

Z = j0.1 PU

TVSI = 1

We demonstrated the triangle math with the 0.9 leading power factor load.

For the 0.9 leading power factor:

1. Solve for β given $\rho\angle\beta$ equals $(0+0.1j)^* S^*$
   a. $\beta = 90 + \mathrm{ArcCos}(0.9) \sim 115.84$ degrees
   b. Use $180 - \beta$ in triangle ~ 64.158 degrees
2. V_u equals V_l at the apex of PV curve. Therefore the associated voltage angles are equal.
   a. $\phi u = \phi l = -57.921$ degrees
      i. [180 – 64.158] / 2
3. Use SAS triangle math to solve the triangle since we have a side with E = 1, and two angles ϕ_u^*, and $180 - \beta$. We used a HP Prime calculator triangle solver application to solve for V_u and V_l.
   a. $V_u = 0.9414@{-57.92}°$ and $V_l = 0.9414@{-57.92}°$
4. Using equation (8) we solved for the real power P.
   a. $V_u * V_l / Z * PF = 7.977$ PU power

We summarize the results in Table 1 for all the scenarios. The voltage V_u and phase angle results compare closely with the graphs in [7].

Given:	Power Factor (PF)

E = 1 PU Z=0.1j PU TVSI = 1	0.9 Lagging	Unity	0.9 Leading
ρ	0.348216	0.5	0.886352
β	64.158°	90.°	115.842°
V_u	0.590	0.707	0.941
ϕu	-32.079°	-45.°	-57.921°
P+jQ	3.1339+j1.5178	5.+j0.	7.9771-j3.8635
Δ	0	0	0

Table 1 – Triangle Voltage Stability Index TVSI

We believe that at the lagging and unity power factors, the system operator would receive ample warning on the voltage V_u magnitude of an impending voltage problem through a state estimator alarm system. However, at the leading power factor scenario, the operator may not be aware of a pending voltage problem since the voltage magnitude looks respectable. Consequently, we believe the operators need to become aware that voltage problems are related to the sending in voltage E, the phase angle ϕ_u, the receiving voltage V_u, and the associated load PF. TVSI would be a relatively simple way to achieve this awareness.

Example 2: System Study Report

In this example we used a sanitized AEP load flow case.

In the example below, we use a PSS®E load flow and a custom PSS®E IPL program to calculate the TVSI on hundreds of circuits after applying thousands of contingencies to each circuit. The PSS®E IPL program utilizes equation (9) to initially identify circuits closest to the critical bifurcation point. We believe Δ is a more sensitive indicator of voltage problems since Δ has the units of voltage whereas the TVSI is a dimensionless ratio of two voltages. However, we still report the results as TVSI because we believe TVSI has a more intuitive feel with the end user's visual attachment to PV curves.

Our basic approach pairs a monitored circuit branch with an associated contingency simulation. The PSS®E IPL program uses the following data:

1. Sending in voltage E
2. Receiving voltage V_u
3. Phase angle ϕ_u between E and V_u.
4. Calculates Δ.
5. Repeat for all monitored circuits and contingencies.

6. Sort by the Δ magnitude and report the associated TVSI results. A lower Δ indicates a potential voltage problem.

From Bus	To Bus	Outage	Stability Index
05RYAN 138.00	05ANU 138.00	05KEVIN 345.0005MARK 345.001 05MARYAM 500.0005MIKE 500.001	0.57
05RYAN 138.00	05DANE 138.00	05KEVIN 345.0005MARK 345.001 05TRISH 345.0005KAIT 345.001	0.51

Table 2: Voltage Stability Index Summary

PSS®E reports the 05RYAN 138kV bus voltage at 126.2-kV for the double contingency in row one. The vector voltage drop across the RYAN -ANU 138-kV path is in the neighbourhood of 72-kVKv and the associated TVSI is 72/126 ~ 0.57.

The program identified a relatively high impedance path from RYAN –ANU. With the application of the double contingency outages, the power flow increases dramatically through the high impedance path which results in a larger vector voltage drop across the path.

Conclusion

In this paper we describe a family of power triangles that enable the power load flow equation to be viewed graphically. In particular, at the critical bifurcation point, we note:

1. The in-phase component of the load voltage equals half of the source voltage, and
2. The square of the receiving voltage magnitude equals the ρ magnitude.

The product of $V_u^* V_l^*$ equals $\rho\angle\beta$.

The voltage V_l equals the voltage V_z.

For leading power factor situations, we believe the system operator needs to become aware that the vector voltage drop across the transmission path is important. AEP has discussed this concern with our system operators.

The triangles describe several voltage stability measures namely TVSI and Δ.

We utilized the Load Flow Voltage Discriminant Triangle in a practical system study report to identify circuits closest to the critical bifurcation point.

We note throughout the paper, the results of the triangles are consistent with the findings in the literature in [1,2,4,5,7,9].

BIBLIOGRAPHY

[1] Ajjarapu, Venkataramana, Computational Techniques for Voltage Stability Assessment and Control, Springer, 2006

[2] Cutsem Thierry V, Voltage Stability of Electric Power Systems, Springer, 2008

[3] Weisstein, Eric W. "Vieta's Formulas." From MathWorld – A Wolfram Web Resource. http://mathworld.wolfram.com/VietasFormulas.html

[4] B. Milosevic and M. Begovic, "Voltage-stability protection and control using a wide-area network of phasor measurements," IEEE Trans. Power Systems, vol 18, no 1, pp 121-127, Feb, 2003.

[5] Anthony B. Morton, A Guide to Steady-State Voltage Stability Analysis,2009

[6] Weisstein, Eric W. "Stewart's Theorem" From MathWorld – A Wolfram Web Resource. http://mathworld.wolfram.com/StewartsTheorem.html

[7] Scott G. Ghiocel and Joe H. Chow, Department of Energy Research Project: PMU-Base Voltage Stability, October 2013, Rensselaer Polytechnic Institute

[8] N.A.M. Ismail, A Comparison of Voltage Stability Indices, 2014 IEEE 8th International Power Engineering and Optimization Conference

[9] Antonio Trias, Fundamentals of the Holomorphic Embedding Load-Flow Method, https://arxiv.org/abs/1509.02421