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Distribution System Operator Market Formulation

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SUMMARY

The deployment of a distribution system operator (DSO) is becoming a necessity as a result of the increasing roles and functionalities of the distribution grid in ensuring an efficient and reliable delivery of electricity to emerging proactive customers. Customer preferences have evolved as they are interested in having more control over their energy use and in conducting transactions with the utility grid. A DSO may efficiently utilize DER to improve system reliability and resiliency, and reduce emissions and greenhouse gasses by resource diversification. Each DSO will be responsible for managing, forecasting, and dispatching intra-DSO resources. However, there are several challenges in the successful deployment of DSOs. For instance, there is a need to identify the necessary infrastructure architecture, substation requirements, and communications technologies in order to implement the DSO. The DSO operational requirements and market function also need to be determined.

In this paper, we wish to demonstrate the benefits of using distribution locational marginal prices (DLMP) based market for distribution systems. To this end, we propose a market structure for distribution systems. More specifically, we make use of a linear implicit power flow model. The linear approximant is sparse, computationally attractive, and preserves the structure of the power flow. The objective function includes the minimization of cost of real power at the substation, distributed generation (DG) costs, and distribution losses. We explicitly represent power system operational constraints such as voltage constraints in the feeder; power flow constraints; thermal limits; real and reactive power injections by load and generators. The outcomes of the market clearing optimization problem provide the DLMPs that include the marginal costs for grid services, distribution losses, among others. We show through the numerical results section that the proposed market structure is tractable and that DLMPs benefit the system by, for instance, they enable price-to-devices mechanism.

KEYWORDS

Distribution Locational Marginal Price (DLMP), Distributed Energy Resources (DER), Distributed Generation (DG), Market Formulation, Optimality conditions

I. INTRODUCTION

The electric power grid is encountering a growing penetration of distributed energy resources (DERs) driven predominantly by growing consumers' preference to environmentally-friendly generation, government incentives, increasing electricity prices, and reduction in cost of DER technologies. While valuable, this growing penetration has the potential to challenge the efficient, reliable, and cost-effective operation of the electric power system if not managed well, hence calling for practical and innovative solutions. One viable solution to address the emerging challenges is the deployment of distribution system operators (DSOs). The DSOs are anticipated to implement numerous applications to improve system visibility and further allow a better utilization of DERs. The deployment of a DSO not only helps cope with the challenges introduced by DERs, but also can help maximize all the benefits associated with DER installations. Many believe that interdependency between DER and DSO is emerging, where for the development of DSOs the deployment of a high number of DERs is necessary and vice versa.

One of the main challenges materializing the emerging DSO construct is to develop the market mathematical formulation of DSOs. The idea of locational marginal prices (LMPs) may be borrowed from the transmission system and implemented in the distribution system for the formulation of distribution locational marginal prices (DLMPs). The DLMPs provide important economic signals that fully reflect both system and market operations at a specified time. In this paper, we include the thermal limits of the lines and voltage limits as constraints of the market. In addition, the DLMPs may be used as control signals to modify the output of DER in order to operate the system in a reliable manner [2]. The DLMP embodies the price of the energy as well as the impacts of the line losses, and the effects of the thermal limits that result in network congestion. In this paper, we propose a decomposition of the DLMP that shows the value associated with the energy component as well as the value of relieving each of the considered constraints. More specifically, we decompose the DLMP into energy, loss, congestion, and voltage support components. The proposed methodology is demonstrated through the IEEE 13-bus system. It is worth noting that there is no unique way of decomposing a DLMP into components and the proposed methodology is a heuristic approach. The reader is referred to [3-7] for further details on the notions of DSO functions and distribution locational margin pricing

II. PRELIMINARIES AND PROBLEM FORMULATION

We formulate the problem using a balanced three-phase system. However, the results may be extended for an unbalanced three-phase network as shown in the numerical results section with the application of the framework on the IEEE 13-bus system.

We consider a power distribution network with n buses. The network topology can be described by a connected tree, the edge set of which is denoted by \mathcal{E} , where $(i, k) \in \mathcal{E}$ if i is connected to k by a line; we write $i \sim k$ if bus i is connected to bus k . Let $V_i = |V_i| \angle \theta_i$ denote bus i voltage, and define the corresponding bus voltage vector $v = [V_1 V_2 \dots V_n]^T \in \mathbb{C}^n$. Let us denote by \mathcal{G} the set of generators and by $P_{g_i}(Q_{g_i})$ the real and reactive power injection of $i \in \mathcal{G}$; P_{g_i} and Q_{g_i} are zero if the bus does not have a generator, i.e., $i \notin \mathcal{G}$. The load at each bus i is denoted by $P_{L_i} + j Q_{L_i}$. Similarly, let $P_i = P_{g_i} - P_{L_i}$ and $Q_i = Q_{g_i} - Q_{L_i}$ denote the active and reactive power injections at bus i respectively, and define the corresponding active and reactive power injection vectors $p = [P_1 P_2 \dots P_n]^T$, $q = [Q_1 Q_2 \dots Q_n]^T$. Let $y_{ik} = g_{ik} - jb_{ik}$, with $b_{ik}, g_{ik} > 0$, denote the admittance of line (i, k) , and let $y_{ii} = jb_{ii}$ denote bus i shunt admittance. The power flow equations based on [8] can be written as $p = A_1|V| + A_2\theta$, $q = A_3|V| + A_4\theta$. The active and reactive power flow through a line (i, k) are denoted P_{ik} and Q_{ik} , respectively. We define the corresponding vectors of active and reactive power flow as $p_\ell = [P_{ik} P_{ki}: \forall (i, k) \in \mathcal{E}]^T$ and $q_\ell = [Q_{ik} Q_{ki}: \forall (i, k) \in \mathcal{E}]^T$, where $p_\ell = \Gamma_1|V| + \Gamma_2\theta$ and $q_\ell = \Gamma_3|V| + \Gamma_4\theta$.

Given the load demands P_{L_i} and Q_{L_i} for $\forall i = 1, \dots, n$ the goal is to select a feasible set of voltages and power supplied by the conventional distributed generation (DG) so that the steady state of the system is optimal. To this end, the following objectives are pursued: (i) Minimization of power losses, (ii) Minimization of the cost of real power procured at the substation, and (iii) Minimization of the DG

costs. The active power losses of line $(i, k) \in \mathcal{E}$ are $L_{ik} = P_{ik} + P_{ki}$. The total active power losses of the system are $L = \sum_{i,k:(i,k) \in \mathcal{E}} L_{ik} = \sum_{i,k:(i,k) \in \mathcal{E}} (P_{ik} + P_{ki}) = \sum_{i=1}^n P_i$, since $P_i = \sum_{k:(i,k) \in \mathcal{E}} P_{ik}$. Let us denote by λ_T the LMP of real power at the bus that the substation is connected to. Then the cost of real power at the substation is $\lambda_T P_{g_1}$, assuming that node 1 is where the substation is located. The cost of distributed generation may be formulated as $\sum_{i \in \mathcal{G}} c_i P_{g_i}$, where c_i is the cost of supplied power for generator $i \in \mathcal{G}$. Now, the market clearing mechanism may be formulated as

$$\begin{aligned}
(1) \quad & \min_{P_{g_i}, Q_{g_i}, V, \theta} \sum_{i=1}^n P_i + \lambda_T P_{g_1} + \sum_{i \in \mathcal{G}} c_i P_{g_i} \\
(2) \quad & p = A_1 |V| + A_2 \theta \leftrightarrow \lambda_p \\
(3) \quad & q = A_3 |V| + A_4 \theta \leftrightarrow \lambda_q \\
(4) \quad & |V_i| \leq |V_i| \leq |\bar{V}_i|, i = 1, \dots, n \leftrightarrow \bar{\mu}_i, \underline{\mu}_i \\
(5) \quad & \underline{P}_{g_i} \leq P_{g_i} \leq \bar{P}_{g_i}, i \in \mathcal{G} \leftrightarrow \bar{\rho}_i, \underline{\rho}_i \\
(6) \quad & \underline{Q}_{g_i} \leq Q_{g_i} \leq \bar{Q}_{g_i}, i \in \mathcal{G} \leftrightarrow \bar{\nu}_i, \underline{\nu}_i \\
(7) \quad & \underline{p}_l \leq p_l \leq \bar{p}_l \leftrightarrow \bar{\xi}, \underline{\xi}
\end{aligned}$$

where $|\bar{V}_i|$ ($|\underline{V}_i|$) is the upper (lower) voltage limit; \bar{P}_{g_i} (\underline{P}_{g_i}) is the upper (lower) limit of real power generation from distributed generator i ; \bar{Q}_{g_i} (\underline{Q}_{g_i}) is the upper (lower) limit of reactive power generation from distributed generator i ; and $\bar{\rho}_i, \underline{\rho}_i$ are the thermal limits of the lines. The dual variables of the associated constraints are demonstrated with the arrows in (2)-(7).

III. MARKET CLEARING MECHANISM OPTIMALITY ANALYSIS

In this section we use the market clearing mechanism presented in Section II in order to derive relationships between the dual variables of the constraints of (3)-(8). The DLMP is denoted λ_p and we wish to decompose it into an energy component λ_r , a losses component λ_l , a congestion component λ_c , and a voltage support component λ_v . As shown in this section we differentiate the definition of the components between the marginal and non-marginal nodes. The Lagrangian function $\mathcal{L} = \mathcal{L}(|V|, \theta, P_{g_i}, Q_{g_i}, \lambda_p, \lambda_q, \bar{\mu}_i, \underline{\mu}_i, \bar{\rho}_i, \underline{\rho}_i, \bar{\nu}_i, \underline{\nu}_i, \bar{\xi}, \underline{\xi})$ of (1) is

$$\begin{aligned}
(8) \quad \mathcal{L} = & \sum_{i=1}^n P_i + \lambda_T P_{g_1} + \sum_{i \in \mathcal{G}} c_i P_{g_i} + \lambda_p^T (-p + A_1 |V| + A_2 \theta) + \lambda_q^T (-q + A_3 |V| + A_4 \theta) + \\
& \sum_{i=1}^n \bar{\mu}_i (|V_i| - |\bar{V}_i|) + \sum_{i=1}^n \underline{\mu}_i (-|V_i| + |\underline{V}_i|) + \sum_{i \in \mathcal{G}} \bar{\rho}_i (P_{g_i} - \bar{P}_{g_i}) + \sum_{i \in \mathcal{G}} \underline{\rho}_i (P_{g_i} - \underline{P}_{g_i}) + \\
& \sum_{i \in \mathcal{G}} \bar{\nu}_i (Q_{g_i} - \bar{Q}_{g_i}) + \sum_{i \in \mathcal{G}} \underline{\nu}_i (Q_{g_i} - \underline{Q}_{g_i}) + \bar{\xi}^T (p_\ell - \bar{p}_\ell) + \underline{\xi}^T (p_\ell - \underline{p}_\ell)
\end{aligned}$$

We denote by $f(|V|, \theta, P_{g_i}) = \sum_{i=1}^n P_i + \lambda_T P_{g_1} + \sum_{i \in \mathcal{G}} c_i P_{g_i}$ the cost function which is the objective of the optimization problem. The Karush–Kuhn–Tucker necessary conditions for the optimum point subject to the feasibility conditions require that

$$\begin{aligned}
(9) \quad & \frac{\partial \mathcal{L}}{\partial P_{g_i}} = \frac{\partial f}{\partial P_{g_i}} - \lambda_{p_i} + \bar{\rho}_i - \underline{\rho}_i = 0, \\
& \nabla_{|V|} \mathcal{L} = \nabla_{|V|} f + A_1^T \lambda_p + A_3^T \lambda_q + \bar{\mu} - \underline{\mu} + \Gamma_1^T (\bar{\xi} - \underline{\xi}) = 0.
\end{aligned}$$

In similar approach as in [9] at the optimum, we partition into two non-intersecting subsets: the nodes that have (distributed) generation units whose MW injection is not at a limit and will be referred to in the following as the marginal nodes (set \mathcal{M}) and nodes with generation fixed at its minimum/maximum

limit, or without generator will be referred to as non-marginal nodes (set \mathcal{M}'). Then we have $\lambda_{p_i}^M = \frac{\partial f}{\partial P_{g_i}}$ and $\lambda_{p_i}^{M'} = \frac{\partial f}{\partial P_{g_i}} + \bar{\rho}_i - \underline{\rho}_i$. Thus, we may partition $\lambda_p = [\lambda_p^{M^T} \lambda_p^{M'^T}]^T$. Then (9) may be rewritten as

$$\begin{aligned}
& \nabla_{|V|} f + [A_1^M \ A_1^{M'}]^T [\lambda_p^{M^T} \ \lambda_p^{M'^T}]^T + A_3^T \lambda_q + \bar{\mu} - \underline{\mu} + \Gamma_1^T (\bar{\xi} - \underline{\xi}) = 0 \Rightarrow \\
(10) \quad & \nabla_{|V|} f + A_1^{M^T} \lambda_p^M + A_1^{M'^T} \lambda_p^{M'} + A_3^T \lambda_q + \bar{\mu} - \underline{\mu} + \Gamma_1^T (\bar{\xi} - \underline{\xi}) = 0 \xrightarrow{\tilde{A}_1 = (A_1^{M'} \ A_1^{M'^T})^{-1} A_1^{M'}} \\
& \lambda_p^{M'} = \tilde{A}_1 \nabla_{|V|} f + \tilde{A}_1 A_1^{M^T} \lambda_p^M + \tilde{A}_1 A_3^T \lambda_q + \tilde{A}_1 (\bar{\mu} - \underline{\mu}) + \tilde{A}_1 \Gamma_1^T (\bar{\xi} - \underline{\xi}) \xrightarrow{\lambda_p^M = \lambda_r 1^M + \lambda_l^M + \lambda_c^M + \lambda_v^M} \\
& \lambda_p^{M'} = \tilde{A}_1 \nabla_{|V|} f + \tilde{A}_1 A_1^{M^T} (\lambda_r^M 1 + \lambda_l^M + \lambda_c^M + \lambda_v^M) + \tilde{A}_1 A_3^T \lambda_q + \tilde{A}_1 (\bar{\mu} - \underline{\mu}) + \tilde{A}_1 \Gamma_1^T (\bar{\xi} - \underline{\xi}) \Rightarrow \\
& \lambda_p^M = \lambda_r 1^{M'} + \lambda_l^{M'} + \lambda_c^{M'} + \lambda_v^{M'}
\end{aligned}$$

With

$$\begin{aligned}
(11) \quad & \lambda_l^{M'} = \lambda_r (1^{M'} + \tilde{A}_1 A_1^{M^T} 1^M) + \tilde{A}_1 A_1^{M^T} \lambda_l^M + \tilde{A}_1 \nabla_{|V|} f \\
& \lambda_c^{M'} = \tilde{A}_1 \Gamma_1^T (\bar{\xi} - \underline{\xi}) + \tilde{A}_1 A_1^{M^T} \lambda_c^M \\
& \lambda_v^{M'} = \tilde{A}_1 (\bar{\mu} - \underline{\mu}) + \tilde{A}_1 A_1^{M^T} \lambda_v^M
\end{aligned}$$

For the marginal units we define $\lambda_p^M = \lambda_r 1^M + \lambda_l^M + \lambda_c^M + \lambda_v^M$, with λ_c^M, λ_v^M derived from (10) for only the marginal buses, and $\lambda_{r_i} = c_i, \lambda_{v_i}^M = \frac{\partial \sum_{i=1}^n P_i}{\partial P_{g_i}}$.

In this section, we presented a heuristic approach on how the DLMPs may be decomposed into four components that can provide signals to the operators and pinpoint which are the system constraining factors and their associated values. In this way, incentives can be given to customers at specific nodes to provide solutions to various problems such as congestion or for settlements among the transacting parties.

IV. NUMERICAL EXAMPLES

In this section, we illustrate the proposed methodology with the IEEE 13-bus system (see Figure 1), which contains four distributed generation resources at buses 633, 646, 680, and 611 [10]. Unless otherwise noted, all quantities are expressed in per unit (pu) with respect to a base power of 5MVA. The LMP at the substation is set to $\lambda_r = 50\$/MW$, and the costs of the distributed generation are: $P_{g_{633}} = 10\$/MW, P_{g_{646}} = 20\$/MW, P_{g_{680}} = 30\$/MW, P_{g_{611}} = 40\$/MW$. In this example, we do not take into account the thermal limits of the lines. We calculate the DLMPs for the marginal and non-marginal units for several values of voltage constraints. More specifically we run two cases: (i) $0.95 \leq |V_i| \leq 1.06, \forall i$ and (ii) $0.99 \leq |V_i| \leq 1.02, \forall i$ for all the three phases.

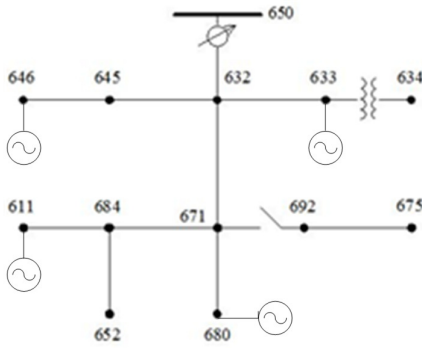


Figure 1: IEEE 13-bus system

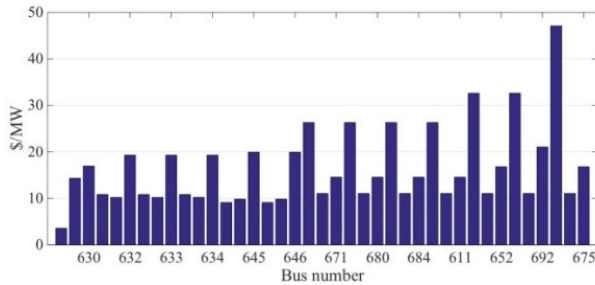


Figure 2: DLMPs of IEEE13-bus system for case (ii)

In case (i) no constraints are binding thus the DLMPs are the same for all the nodes and show the cost of delivering energy to the system. The results of the DLMPs for each of the three phases for each node are $\lambda_{i_A} = 10\$/MW$, $\lambda_{i_B} = 10\$/MW$, and $\lambda_{i_C} = 20\$/MW$ for all nodes for the three phase A, B, and C. However, in case (ii) the voltage constraints are binding forcing the redispatch of generation to obtain a feasible solution of the market clearing mechanism presented in (3). More specifically, the marginal generator is located in bus 646. In Figure 3, the DLMPs for case (ii) are depicted for the entire system. We can notice now that each phase and each bus have different values of DLMPs. The reason is that some voltages constraints were binding. Now we use the decomposition methodology that was presented in equations (9)-(11) in Section III. In Figure 3, we show the energy component of the DLMP decomposition, which is the same for all the buses in the system. However, as we may see in Figure 4, where the voltage support component of the DLMP is depicted, the values are different at each node and phase. These values may be interpreted as the value/cost of enforcing the voltage constraint at these nodes and may be used to quantify the benefit of providing voltage support at specific nodes. We may notice that the value of voltage support is higher as the distance of the bus for the substation is longer.

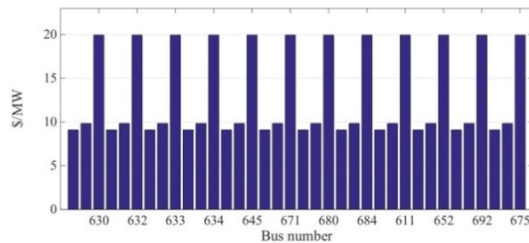


Figure 3: DLMP energy components of IEEE 13-bus of IEEE 13-bus system for case (ii)

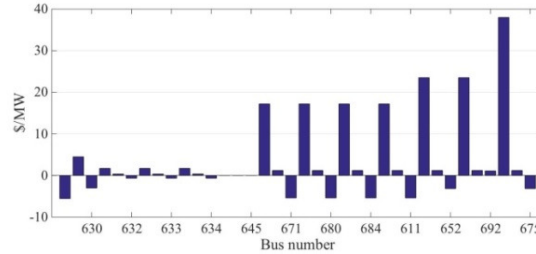


Figure 4: DLMP voltage support components of IEEE 13-bus system for case (ii)

V. CONCLUDING REMARKS

In this paper, we presented the power flow model of a distribution system and formulated the market clearing mechanism of a DSO. More specifically, we defined the DLMP of a system, which shows the marginal cost of supplying one more MW of energy in a specific node. We used the optimality conditions of the market clearing mechanism to derive relationships between the DLMPs and the dual variables of other constraints such as thermal limits or voltage constraints. More specifically the DLMP was decomposed into four components: the energy, the losses, the congestion, and the voltage support components. We demonstrated the proposed framework with the IEEE 13-bus system. This framework may be used in the construction of policies that enable the deployment of DERs and incentivize customers to provide grid services to the system.

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