21, rue d'Artois, F-75008 PARIS
http : //www.cigre.org

CIGRE US National Committee
2015 Grid of the Future Symposium

Real Time Phasor Data Processing using Complex Number Analysis Methods

R.A. DE CALLAFON[1], C.H. WELLS[2]
[1]University of California, San Diego
[2]OSIsoft LLC

USA

SUMMARY

Event detection and analytic calculations in both Wide Area Measurement Systems (WAMS) and in distribution systems including Phasor Measurement Unit (PMU) based microgrid control systems often use the independent components of the phasors (e.g. frequency or Voltage) rather than the vector representation of a phasor. This paper introduces the use of the discrete time complex Fourier Transform based on complex number phasor data to track the real-time dynamic behaviour of the power grid. The specific property in which real and imaginary valued signals are mapped to complex and complex conjugate parts of the Fourier Transform can be used to compute the Fourier Transform of two signals simultaneously. One signal is placed in the real part of the time domain, while the other is place in the imaginary part. The notion of computing Fourier Transforms of signals that contains both a real part and an imaginary part carry over to PMU data to detect disturbances that may in both the real/imaginary or amplitude/phase part of the phasor.

The approach of complex Fourier Transform analysis applies to both the transmission and distribution grids providing insight into the stability of the grid as well as it operating stability margin and is especially important in distribution systems with high levels of renewable generation combined with intermittent electric vehicle charging loads. Fourier transforms are often used in Oscillation detection programs to determine the grid modes in both steady state and under transient conditions. The modes exist in both the Western and the Eastern Interconnections and occur continuously at varying magnitudes. This paper demonstrates the value of treating the PMU as true vectors to obtain more information about the state of the grid. We demonstrate this by showing how using this state information leads to improved event detection and moving window FFT analysis using the complex phasors as input. For oscillation detection we use either the rate of change in real and imaginary parts or the rate of change in amplitude and angle (frequency) of the phasor as input to complex moving window FFTs. We demonstrate the value of using complex numbers for these analyses on actual grid PMU data.

cwells@osisoft.com

KEYWORDS

Phasor Measurement Unit; Event Detection; Complex Algebra; Fourier Transform;

INTRODUCTION

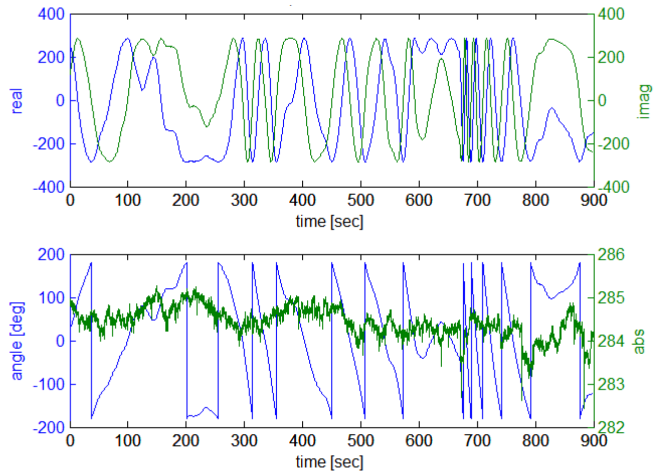

Figure 1: Illustration of rectilinear (top) and polar (bottom) phasor data representation for a PMU measurement exhibiting a power oscillation disturbance just before t=700sec.

Phasor Measurement Units (PMU) are now widely used in both Wide Area Measurement Systems (WAMS) and in distribution systems for time synchronized measurements of current, voltage, frequency and power flow [1,2]. By definition, phasors are complex numbers whether represented in rectangular or polar coordinate systems. The IEEE C37.118 standard for phasor measurement units define the phasor outputs as being reported in either rectangular or polar coordinates as a vector operator consisting of two numbers. The vectors are reported at time intervals referenced to the GPS UTC top of the second time alignment pulse with absolute time accuracy of better than one microsecond. The measurement itself is defined as a “vector error” rather than the traditional “scalar error”. The PMUs output phasor data is rated at better than one percent vector error. This implies that the time stamp for each measurement cannot exceed 26 microseconds even with perfect precision of the real and imaginary parts of the vector. Thus typically with measurement precisions of less than 16 bits, the actual time error must be less than 26 microseconds.

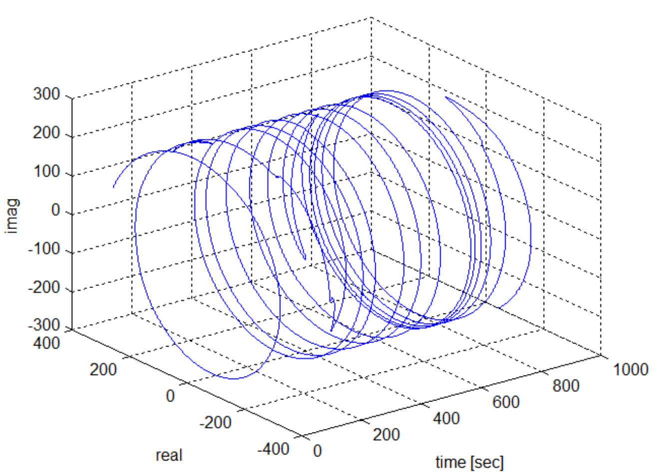

Figure 2: Illustration of time progression of the phasor in a 3D plot with time, real and imaginary part of the phasor.

Although the representation of phasor data in rectilinear or polar coordinates is theoretically equivalent, the representation in polar coordinate system has an additional challenge: the four quadrant phase angle may be discontinuous and jump between ± 180 degrees. Any analysis such as Fourier Transform analysis or real-time filtering using angle phasor data must take this into account by either being robust against this this discontinuity or include a phase unwrapping that is robust against noise on the phase measurements. In contrast, the real and imaginary components of the phasor data will not exhibit such discontinuous behaviour and does not have to be subjected to an unwrap function to maintain continuity of the time series. An illustration of the difference between the rectilinear and polar coordinate time series data of actual PMU data is illustrated in Figure 1. Plotting the same rectilinear coordinates in a three dimensional plot as done in Figure 2 illustrates the continuous behaviour in both real/imaginary but also amplitude/phase behaviour of the phasor, but requiring an unwrapping function of the angle. Although the angle does take into account both the real and imaginary part of the PMU data, it is advantageous to process and real and imaginary part of the phasor directly without the need of phase unwrapping to avoid discontinuity in the time series. This can be done by a complex Fourier analysis that process the phasor as a complex vector.

COMPLEX FOURIER TRANSFORM

A standard Discrete Fourier Transform (DFT) maps N points in the time domain of a real valued signal $x(k)$ into N points in the frequency domain of a complex valued signal $X(n)$ via the (normalized) DFT

$$X(n) = \sum_{k=1}^{N} x(k)e^{-j2\pi kn/N}$$

With Euler's formula

$$e^{j\omega} = \cos\omega + j\sin\omega$$

it is immediately clear that $X(0)$ and $X(N/2)$ are real valued and $X(n+N/2) = X(n)^*$ where $(\cdot)^*$ denotes the complex conjugate. This means that for real valued signals $x(k)$, the Fourier Transform $X(n)$ for $n > N/2$ does not introduce any new (complex) information and it suffices to plot $X(n)$ only up to $n = N/2$ (Nyquist frequency).

The situation is slightly different when $x(k)$ is actually a complex valued signal (such as a phasor). To explain this in a bit more detail, let $x(k) = x_r(k) + j \cdot x_i(k)$ and let $X_r(n)$ and $X_i(n)$ be the DFT of the real-valued signal $x_r(k)$ and $x_i(k)$ respectively. With $X_r(n)$ and $X_i(n)$ being a DFT (complex number) of a real valued signal, we may write two equations for each of them:

$$X_r(n) = a + jb \quad X_r(n+N/2) = a - jb$$
$$X_i(n) = c + jd \quad X_i(n+N/2) = c - jd$$

With $x(k) = x_r(k) + j \cdot x_i(k)$, the DFT $X(n)$ of $x(k)$ satisfies

$$X(n) = X_r(n) + j \cdot X_i(n) = (a-d) + j(c+b)$$

whereas

$$X(n+N/2) = X_r(n+N/2) + j \cdot X_i\left(n + \frac{N}{2}\right) = (a+d) + j(c-b)$$

The last two equations illustrate two important properties of the complex DFT (where $x(k)$ is a complex number) [3]:

1. The DFT $X(n)$ will not have the same complex conjugate property as the individual DFTs $X_r(n)$ and $X_i(n)$, e.g. $X(n+N/2) \neq X(n)^*$.
2. However, the DFT $X(n)$ will be a combination of the real and imaginary parts of the individual DFTs $X_r(n)$ and $X_i(n)$, e.g. $Re\{X(n)\}$ equals $Re\{X_r(n)\} - Im\{X_i(n)\}$.
3. Moreover, the individual DFTs $X_r(n)$ and $X_i(n)$ can be reconstructed from the DFT $X(n)$ and $X(n+N/2)$, e.g. $Re\{X_R(n)\} = 1/2Re\{X(n)\} + 1/2Re\{X(n+N/2)\}$.

This also illustrates that one can compute (reconstruct) the DFTs from two individual signals from a single complex DFT, where one signal is placed in the real part of the time domain, while the other is place in the imaginary part. This result is immediately applicable to phasor data, already given as complex valued data.

APPLICATION TO PHASOR DATA

For the illustration of the complex DFT to phasor data, consider again the phasor data depicted earlier in Figure 1. The top figure represents the value of the real and imaginary part of the A phase voltage phasor of a three phase network. The bottom figure represents the magnitude and the phase of the A phase voltage phasor. The bottom figure confirms that the magnitude (rms) of the Phase A voltage is nearly constant over the time range. This also confirms that there is sparse information in the voltage magnitude measurement, but the real and imaginary representation shows more dynamic information about the grid.

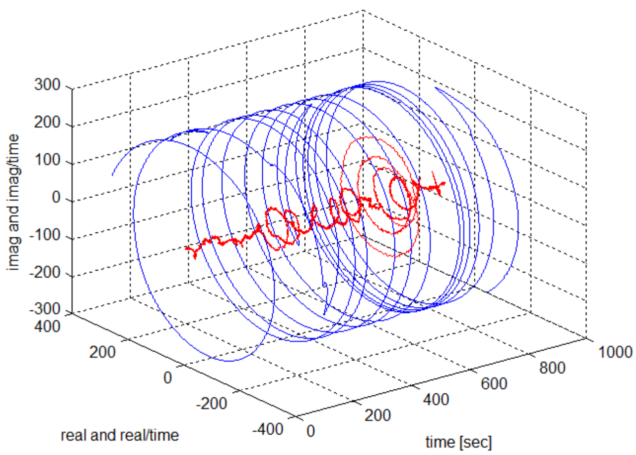

Figure 3: Illustration of time progression (blue) and differential change (red) of the phasor in a 3D plot with time, real and imaginary part of the phasor.

For detecting oscillations in the power grid, we do not consider the absolute measurements of the real and imaginary part of the phasor, but changes in the real and imaginary part. Based on the discrete time measurement of the real $P_r(k)$ and the imaginary $P_i(k)$ part of the phasor $P(k) = P_r(k) + j \cdot P_i(k)$, the differential change is defined as

$$D(k) = \frac{1}{\Delta t} P(k) - P(k-1)$$

where $1/\Delta t = 30\text{Hz}$ for the PMU data used in the paper here. The time progression of the differential change in the real/imaginary part of the PMU data is plotted against time in Fig. 3.

Creating two dimensional time plots of the differential change of the phasor data directly illustrates the presence of a disturbance in the phasor data, as illustrated in Fig. 4. For comparison purposes, the same polar phasor data that was depicted in Fig.1 has been included in Fig. 4. It is clear that the differential display of the rectilinear coordinates of the phasor data directly illustrate the presence of a disturbance on the electricity grid and observed in the measured PMU data.

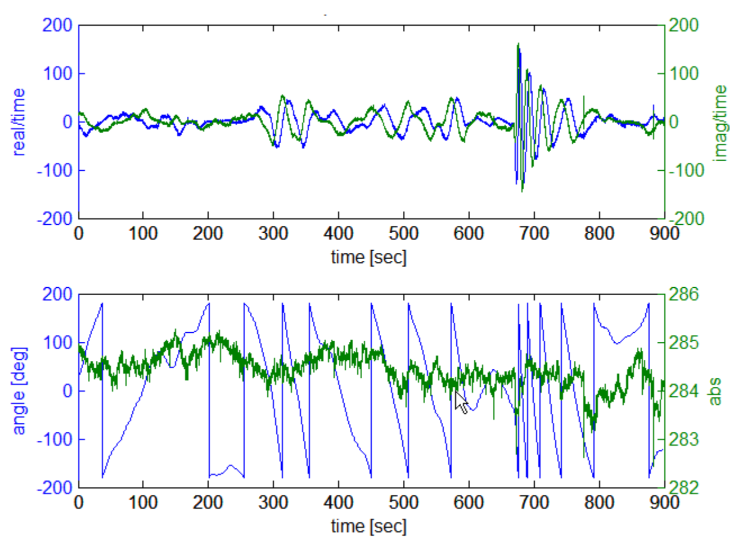

Figure 4: Differential change in real/imaginary part of the phasor data (top) and polar (bottom) phasor data, exhibiting a power oscillation disturbance before t=700sec.

The presence of the disturbance becomes even more clear when a moving finite time complex DFT is computed over the differential rectilinear coordinates of the phasor data. The water plot of a 512 point DFT is given in Fig. 5 where it can be seen that that spectrum of the phasor indeed spikes when the disturbance occurs close to t=700.

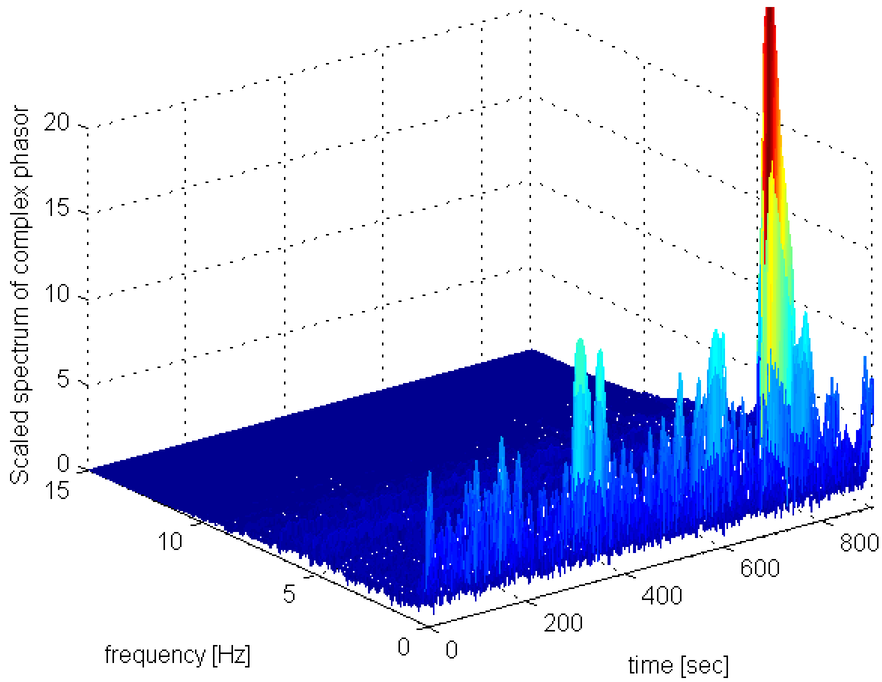

Figure 5: Finite point (N=512) moving average of spectrum of complex DFT based on the differential change in real/imaginary part of the phasor data given earlier in Fig. 4.

CONCLUSIONS

A phasor contains two pieces of information available in either real/imaginary or magnitude/phase-angle. Typically only one of the pieces of information is used (monitoring magnitude or phase/frequency) and improvement in data processing can be achieved by processing information together. In this paper it is shown how this can be done via single Discrete Fourier Transform (DFT) that processes the phasor as a complex number. After the DFT calculation, the Fourier transform of the individual pieces of information could be separated by an decomposition that exploits the complex conjugate properties over different frequency ranges.

BIBLIOGRAPHY

[1] J.Y.Cai. Z. Huang; J. Hauer, K. Martin. "Current Status and Experience of WAMS-Implementation in North America" Transmission and Distribution Conference and Exhibition: Asia and Pacific, (2005)

[2] Phadke, A.G. "Synchronized phasor measurements-a historical overview". Transmission and Distribution Conference and Exhibition (2002)

[3] Smith, S. “The Scientist and Engineer's Guide to Digital Signal Processing”, (1998).