Robustness Adjustment of Two-Stage Robust Security-Constrained Unit Commitment

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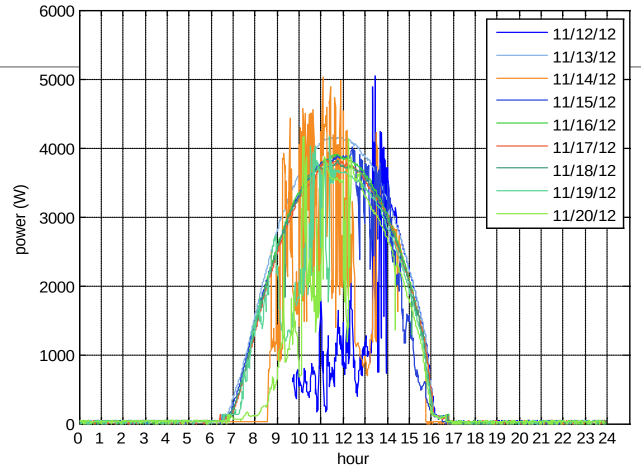
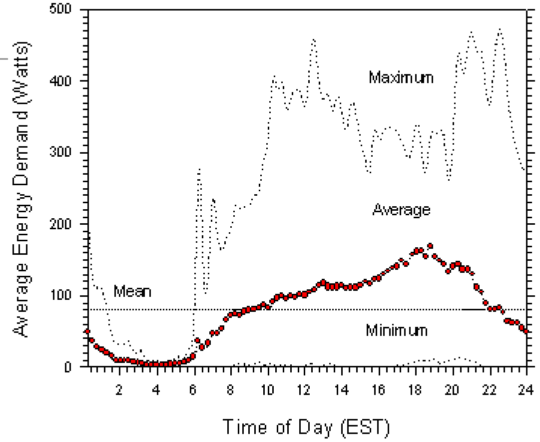
Challenges in smart grid

- Integration of renewable energy and prediction of customer demand bring up uncertainty:



Data from Simrall solar power project, MSU



Daily range electric load profile (*www.fsec.ucf.edu*)

- Reasons for uncertainty include:
  - Seasonality-effects of the days of the week and special days, such as holidays
  - Price-the lower the energy price, the higher the energy consumption
  - Size of household-more people at home, means more energy consumption
  - Energy use patterns-the level of people’s activities
  - Weather-the hotter it is, the more energy is used

The uncertainty from intermittent and unpredictable solar power generation, as well as imprecise customer load, causes challenges for reliable energy supply.

Stochastic optimal operation

- Properties of stochastic programming (SP)
  - Explicitly incorporates a probability distribution of the uncertainty;
  - Relies on pre-sampling discrete scenarios of the uncertainty realizations;
  - Provides probabilistic guarantees to the system reliability with stochastic solutions;
  - Provides the optimal strategy (policy) for the realization of uncertainty.

- Disadvantages of SP
  - Difficult to identify an accurate probability distribution of the uncertainty in SP;
  - Only provide probabilistic guarantees to the system reliability with stochastic solutions ;
  - Becomes extremely large with the increase of the number of stages.

Robust optimal operation

- Properties of robust optimization

  - Only requires moderate information of the underlying uncertainty, such as the mean and the range of the uncertain data;

  - Provides absolute guarantee with an optimal solution that immunizes against all realizations of the uncertain data within a deterministic uncertainty set.

Adaptive robust optimization

$$\underset{y}{Min}\, cy + \underset{u}{Max}\, \underset{x}{Min}\, dx$$

First stage variables

First-Stage Constraints

$$Ay \geq b$$

Second-Stage Constraints

$$Dy + Ex + Ru \geq f$$

Second Stage Variables

$$\underline{x} \leq x \leq \overline{x}$$

$$y \in Z_{+} \qquad x \in R_{+}$$

Robust formulation

$$\underset{I}{Min} \sum_{i=1}^{NG}\sum_{t=1}^{NT} a_i I_{it} + SUD_{it} + \underset{D}{Max}\ \underset{P,PC}{Min} \sum_{i=1}^{NG}\sum_{t=1}^{NT} b_i P_{it} + \sum_{j=1}^{ND}\sum_{t=1}^{NT} r_j PC_{D,jt}$$

Start up and shut down cost

$$SUD_{it} \geq su_i (I_{it} - I_{i,t-1})$$

$$SUD_{it} \geq sd_i (I_{i,t-1} - I_{it})$$

System power balance

$$\sum_{i=1}^{NG} P_{it} = \sum_{j=1}^{ND} D_{jt} - PC_{D,jt}$$

Generation capacity limits

$$P_i^{min} I_{it} \leq P_{it} \leq P_i^{max} I_{it}$$

Load shedding limits

$$0 \leq PC_{D,jt} \leq D_{jt}$$

Power flow limits

$$PL^{min} \leq SF(K_p P_G - K_D(D - PC_D)) \leq PL^{max}$$

Unit ramping up and down limits

$$P_{it} - P_{i,t-1} \leq UR_i[1-(I_{it} - I_{i,t-1})] + SP_i(I_{it} - I_{i,t-1}) + P_i^{max}(1 - I_{it})$$

$$P_{i,t-1} - P_{it} \leq DR_i[1-(I_{i,t-1} - I_{it})] + DP_i(I_{i,t-1} - I_{it}) + P_i^{max}(1 - I_{i,t-1})$$

Unit minimum ON/OFF time limits

$$\sum_{n=k}^{k+T_{on,i}-1} I_{in} \geq T_{on,i} \cdot (I_{ik} - I_{i,k-1}), \forall j, \forall k \quad \sum_{n=k}^{T} [I_{in} - (I_{ik} - I_{i,k-1})] \geq 0, \forall j, \forall k$$

$$\sum_{n=k}^{k+T_{off,i}-1} (1 - I_{in}) \geq T_{off,i} \cdot (I_{i,k-1} - I_{i,k}), \forall j, \forall k \quad \sum_{n=k}^{T} [1 - I_{in} - (I_{i,k-1} - I_{i,k})] \geq 0, \forall j, \forall k$$

- I_{it}: unit commitment status
- P_{it}: power dispatch
- SUD_{it}: cost of start up and shut down
- su_i: start up cost
- sd_i: shut down cost
- D_{jt}: load demand
- $pc_{D,jt}$: load shedding
- p_i^{min}, p_i^{max}: power capacity limits
- UR_i, DR_i: ramp up/down limits
- SP_i, DP_i: start up/ shut down generation
- $T_{on,i}, T_{off,i}$: On/ OFF time of unit i at time t

Uncertainty modeling

- The uncertain load is modeled with the nominal value centered at $\bar{D}_{jt}$ and the deviaion at $\hat{D}_{jt}$

$$D_{jt}^{\min} \le D_{jt} \le D_{jt}^{\max}$$

$$\bar{D}_{jt} = (D_{jt}^{\max} + D_{jt}^{\min}) / 2$$

$$\hat{D}_{jt} = (D_{jt}^{\max} - D_{jt}^{\min}) / 2$$

- The direction of the uncertainty range can be controled by the sign variable z_{jt}, and the amplitude of the uncertainty range is adjusted by the scaled deviation η_{jt}

$$D_{jt} = \bar{D}_{jt} + \hat{D}_{jt} \cdot \eta_{jt} \cdot z_{jt}$$

- The uncertainty model can be expressed as following by introducing binary variables $\beta_{jt}^{\pm}$ for z_{jt}

$$D_{jt} = \bar{D}_{jt} + \hat{D}_{jt} \cdot \eta_{jt} \cdot (\beta_{jt}^{+} - \beta_{jt}^{-})$$

$$z_{jt} = \beta_{jt}^{+} - \beta_{jt}^{-}$$

$$\beta_{jt}^{+} + \beta_{jt}^{-} \le 1$$

$$\beta_{jt}^{+}, \beta_{jt}^{-} \in \{0,1\}$$

Robustness adjustment

- The amplitude of the uncertainty band at each bus is controlled by the parameter Γ_{jt}

$$\eta = \{\eta_{jt} : \eta_{jt} \le \Gamma_{jt}, \eta_{jt} \le 1\}$$

- The bus robustness Γ_t is defined to control the number of buses reaching the worst case at a certain hour

$$Z_t = \{z_{jt} : \sum_j |z_{jt}| \le \Gamma_t, |z_{jt}| \le 1\}$$

- The hourly robustness parameter Γ_j is referred to the period for each bus to reach the worst case during a day

$$Z_j = \{z_{jt} : \sum_t |z_{jt}| \le \Gamma_j, |z_{jt}| \le 1\}$$

These three control patterns can be applied separately or collaborately with each other to improve the flexibility of the proposed model.

Benders decomposition

- **The Master Problem**

- dealing with the binary variables relating to unit commitment decisions

$$Min \quad F = \sum_{i=1}^{NG} \sum_{t=1}^{NT} a_i I_{it} + SUD_{it} + \alpha_l$$

- **Subject to** the start up and shut down cost, and minimum ON/OFF limits.
- The first-stage decisions will be forwarded into the second-stage problem
- The first-stage problem will interact with the second-stage problem through the optimal cut

Benders decomposition

- **The Slave Problem**

$$\underset{D}{Max}\,\underset{P,PC}{Min}\quad \sum_{i=1}^{NG}\sum_{t=1}^{NT} b_i P_{it} + \sum_{j=1}^{ND}\sum_{t=1}^{NT} r_j PC_{D,jt}$$

  - sub-problem of adjustable robust SCUC should meet the power dispatch and load shedding requirements related to the integer unit commitment decisions at the first stage
  - By introducing the dual theory in the slave problem

$$\begin{aligned}\Re(\widehat{I}) = \underset{D,\pi}{Max}\quad & \sum_{i=1}^{NG}\sum_{t=1}^{NT} -P_i^{\max} I_{it}\cdot\pi_{1,it} + P_i^{\min} I_{it}\cdot\pi_{2,it} - \sum_{i=1}^{NG}\sum_{t=1}^{NT}\hbar(\pi_{3,it}) + \lambda\!\!\!^{-}(\pi_{4,it}) \\ & + \sum_{t=1}^{NT}\sum_{j=1}^{ND} D_{jt}\{\pi_{5,t} - \pi_{8,jt} + \sum_{k=1}^{NL} SF_k K_{D,j}(-\pi_{6,kt} + \pi_{7,kt})\} + \sum_{t=1}^{NT}\sum_{k=1}^{NL} -\pi_{6,kt}\cdot PL^{\max} + \pi_{7,kt}\cdot PL^{\min}\end{aligned}$$

  - The optimal cut is generated as

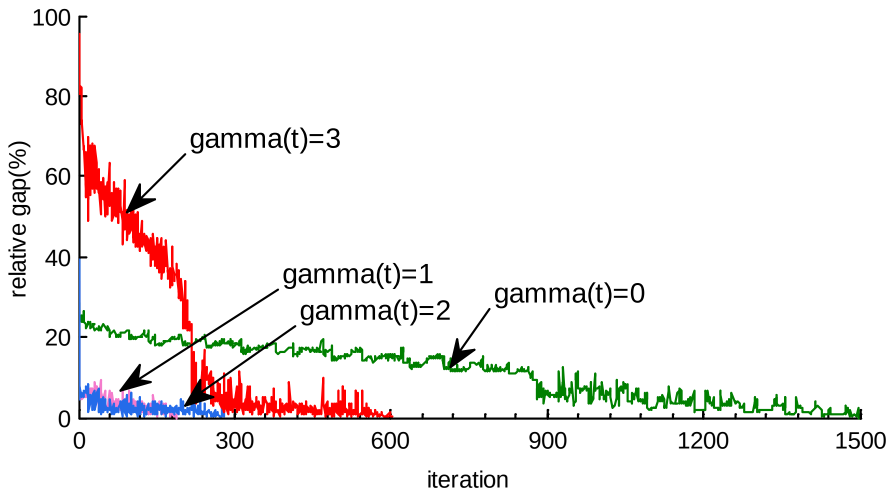
$$\begin{aligned}\alpha \geq & \sum_{i=1}^{NG}\sum_{t=1}^{NT} -P_i^{\max} I_{it}\cdot\widehat{\pi}_{1,it} + P_i^{\min} I_{it}\cdot\pi_{2,it} - \sum_{i=1}^{NG}\sum_{t=1}^{NT}\hbar(\pi_{3,it}) + \lambda\!\!\!^{-}(\pi_{4,it}) + \sum_{t=1}^{NT}\sum_{j=1}^{ND}\{D_{jt}^{*}\pi_{5,t} - D_{jt}^{*}\pi_{8,jt}\} \\ & + \sum_{t=1}^{NT}\sum_{k=1}^{NL}\widehat{\pi}_{6,kt}\cdot\{-PL^{\max} - \sum_{j=1}^{ND} SF_k\cdot K_{D,j}\cdot D_{jt}^{*}\} + \sum_{t=1}^{NT}\sum_{k=1}^{NL}\pi_{7,kt}\cdot\{PL^{\min} + \sum_{j=1}^{ND} SF_k\cdot K_{D,j}\cdot D_{jt}^{*}\}\end{aligned}$$

Impact of Uncertainty Range

Computation results with various uncertainty range

Case	Uncertainty range	Iteration (#)	Cost ($)	Time elapsed (s)
1	[1,1]	2587	9.4626e+04	2.3680e+04
2	[0.9,1.1]	1272	1.0537e+05	3.9719e+03
3	[0.85,1.15]	926	1.1113e+05	1.7273e+03
4	[0.75， 1.25]	604	1.2341e+05	1.3743e+03
5	[0.5, 1.5]	3422	load shedding applied	1.7585e+004

Impact of Bus Robustness



Impact of bus robustness on computation process

Impact of Hourly Robustness

Computation results with variable Hourly robustness and fixed uncertainty range [0.75, 1.25], $\Gamma_3 = \Gamma_4 = 24$

gamma Γ_2	worst-load hours	Iteration (#)	Cost ($)	Time elapsed (s)
24	1-24	604	1.2341e+05	1.3743e+03
18	5, 7-9, 11-24	354	1.2296e+05	1.2635e+03
12	1,6, 12-21	214	1.2165e+05	1.9230e+03
6	1, 15-19	318	1.1984e+05	1.7633e+03
0	--	267	1.1730e+05	799.0703

Computation results with variable Hourly robustness and fixed uncertainty range [0.75, 1.25], $\Gamma_2 = \Gamma_3 = 24$

gamma Γ_4	worst-load hours	Iteration (#)	Cost ($)	Time elapsed (s)
24	1-24	604	1.2341e+05	1.3743e+03
18	1,8-24	332	1.2137e+05	1.6745e+03
12	1,6,12-14,16-22	273	1.1923e+05	1.0847e+03
6	6,8,13-15,18	299	1.1690e+05	1.4471e+03
0	--	210	1.1113e+05	618.2621

Computation results with variable Hourly robustness and fixed uncertainty range [0.75, 1.25],$\Gamma_2 = \Gamma_4 = 24$

gamma Γ_3	worst-load hours	Iteration (#)	Cost ($)	Time elapsed (s)
24	1-24	604	1.2341e+05	1.3743e+03
18	1,6,9-24	304	1.2190e+05	1.1420e+03
12	6,12-22	332	1.1944e+05	1.5567e+03
6	14-19	172	1.1620e+05	1.0979e+03
0	--	224	1.1113e+05	917.9231