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Trajectory Sensitivity Analysis as a Means of Performing Dynamic Load Sensitivity Studies in Power System Planning

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SUMMARY

One of the most important aspects of time-domain simulations for power system planning studies is the modeling of load. The North American Reliability Corporation (NERC) Transmission Planning (TPL) Standard -001-4 states that "System peak load levels shall include a load model which represents the expected dynamic behavior of loads that could impact the study area, considering the behavior of induction motor loads." This further emphasizes the need to consider detailed load modeling from the perspective of present reliability standards in North America. One of the major challenges with load modeling is determining the exact composition of the load and establishing reasonable aggregated load models. The process of developing such aggregated load models for transmission planning studies inherently introduces significant uncertainties and approximations into the model. Thus, it is important to consider methods for performing load sensitivity studies in a more systematic way. This paper presents some initial research results for using a methodology known as trajectory sensitivity analysis for performing load sensitivity studies.

KEYWORDS

Load modeling, Trajectory Sensitivity, Transient Stability

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1. INTRODUCTION

One of the most important aspects of time-domain simulations for power system planning studies is the modeling of load [1]. Many utilities and reliability entities worldwide are increasing giving renewed attention to the subject of load modeling and in particular dynamic load models for time-domain stability studies in planning [1], [2]. The subject of load modeling is not a new one, and much research and work has been done in the past relative to load models [3], [4], [5], [6] and [7]. Recently, however, a significant body of work has been done in the Western Electricity Coordinating Council (WECC) Load Modeling Task Force which lead to the development of a new composite load model now being adopted in WECC and starting to get used elsewhere in North America [8]. The significant contribution of this work was in the development of the residential air-conditioner compressor motor model, which was the result of a collaborative effort of laboratory testing of such loads [9]. The North American Reliability Corporation (NERC) Transmission Planning (TPL) Standard -001-4 states that "System peak load levels shall include a load model which represents the expected dynamic behavior of loads that could impact the study area, considering the behavior of induction motor loads." As the NERC standards become enforced in the near future, utilities in North America will need to adopt dynamic load model structures similar to the WECC composite load model (*cmpldw*).

In transmission planning studies, loads are modeled as single aggregated load models at a substation node in the transmission model. This is true of the WECC *cmpldw* model as well as all the other models in the references mentioned above. Thus, a significant remaining challenge with load modeling is determining the exact composition of the load and establishing reasonable parameters for the aggregated load models. There have been many approaches proposed for determining the parameters of the aggregated model including such techniques as measurement based model parameter derivation [10]. However, all these methods have significant limitations (e.g. capturing useful event for parameter derivation during peak load hours/seasons) and thus the process of developing such aggregated load models for transmission planning studies inherently introduces significant uncertainties and approximations into the model. Due to these shortcomings, and other reasons [11], it is important to consider methods for performing load sensitivity studies in a more systematic way.

This paper presents some initial research results for using a methodology known as trajectory sensitivity analysis for performing load sensitivity studies. The idea of the approach is a follows: (i) a single simulation of a critical outage under a peak load scenario is performed on the power system model, (ii) all the sensitivities relative to each load model parameter are calculated using the power flow Jacobian at each step of the time domain solution, and (iii) sensitivities to the system response trajectory (e.g. how the voltage at a major bus recovers) to variations in the load model parameters are calculated from the sensitivities without the need for re-simulating the case. As a result the process of sensitivity analysis becomes more systematic and also offers the potential for time savings

2. THE TRAJECTORY SENSITIVITY ANALYSIS METHOD

A power system can be represented by a set of differential algebraic equations [12]

$$\dot{x} = f(x, y, \lambda) \tag{1.1}$$

$$g^{-}(x, y, \lambda) = 0 \text{ for } y_k < 0 \tag{1.2}$$

$$g^{+}(x, y, \lambda) = 0 \text{ for } y_k \geq 0 \tag{1.3}$$

where, x represents the network dynamic states, y represents the network algebraic variables and λ represents the parameters of the system of interest. For the purpose of this paper, λ represents the load model parameters. The trajectory sensitivities to a change in a parameter λ are given by $\frac{\partial x(t)}{\partial \lambda}$ and

$\frac{\partial y(t)}{\partial \lambda}$; the first order partial derivatives. The predicted trajectories can be the calculated by a first order approximation as

$$x(t)_{pred} = x(t)_{old} + \frac{\partial x}{\partial \lambda}\Delta\lambda \tag{1.4}$$

$$y(t)_{pred} = y(t)_{old} + \frac{\partial y}{\partial \lambda}\Delta\lambda \tag{1.5}$$

A detailed description of trajectory sensitivity analysis and the computation method can be found in [12] and [13]. Calculation of the sensitivities involves using the power flow Jacobian at each step of the time domain solution. The power flow Jacobian is available as a by-product of a traditional time domain simulation routine, which incorporates an implicit integration algorithm. The runtime availability of the power flow Jacobian reduces the computation effort in evaluating the sensitivities and it can be done simultaneously while performing a time domain simulation. Since the sensitivities of different parameters are independent of each other, these can be evaluated in parallel using some parallel computing architecture, enabling additional savings in time. Reference [14] provides details of trajectory sensitivity analysis using cluster computing.

3. THE LOAD AND SYSTEM MODEL

Since the approach uses an implicit integration technique, and for the sake of ensuring maximum flexibility during the R&D phase, this work was done in MATLAB® using the PSAT[1] package. A summer peak case for the WECC system was imported into the PSAT environment in MATLAB®. The WECC *cmpldw* [8] model was developed in MATLAB® and all the loads in the system were represented using this model. The initial load composition was set based on the WECC load model task force spreadsheet with estimates of load composition based on climatic regions. The rest of the load model parameters were set to the default settings. The WECC *cmpldw* model structure is shown in Figure 1. A WECC model was used in order to ensure that the test case closely represented a practical real-life scenario. However, it must be understood that this work constitutes research and hence none of the results shown here necessarily reflect actual system performance.

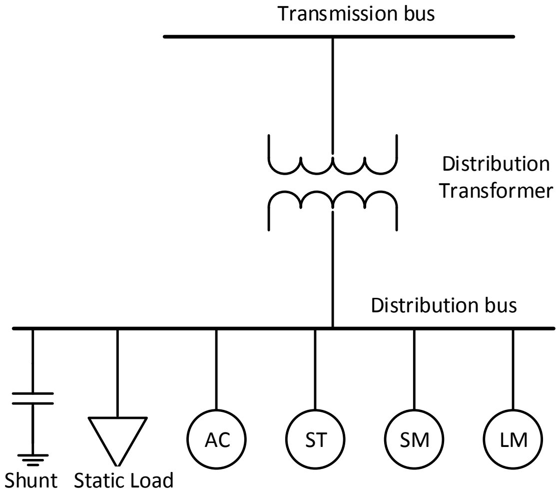

Figure 1: Composite load model in MATLAB based PSAT

4. APPLYING TRAJECTORY SENSITIVITY

Trajectory sensitivity analysis was used to study the effect of change in load composition at different buses on the system algebraic and state variables following a single disturbance. A WECC 2012 summer peak system model was used for this work. A three-phase fault was applied on a major 500

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[1] http://faraday1.ucd.ie/psat.html

kV line. The fault is cleared by opening the line after 5 cycles. The sensitivity of voltage and frequency to percentage changes in the air-conditioning (AC) load at 20 buses were studied. ACs were observed to stall at these 20 buses and hence these 20 buses were selected to study the effect of change in load composition.

Let K_p designate the percentage of AC load at each load bus. Then, ΔK_p is the percentage change in AC load at each bus. Figure 2 shows the actual and predicted bus voltages at bus 1 for a ΔK_p of 5 percent at the 20 study buses. It can be seen that the predicted and actual voltages follow closely. Bus 1 is a major system load bus electrically close to the disturbance.

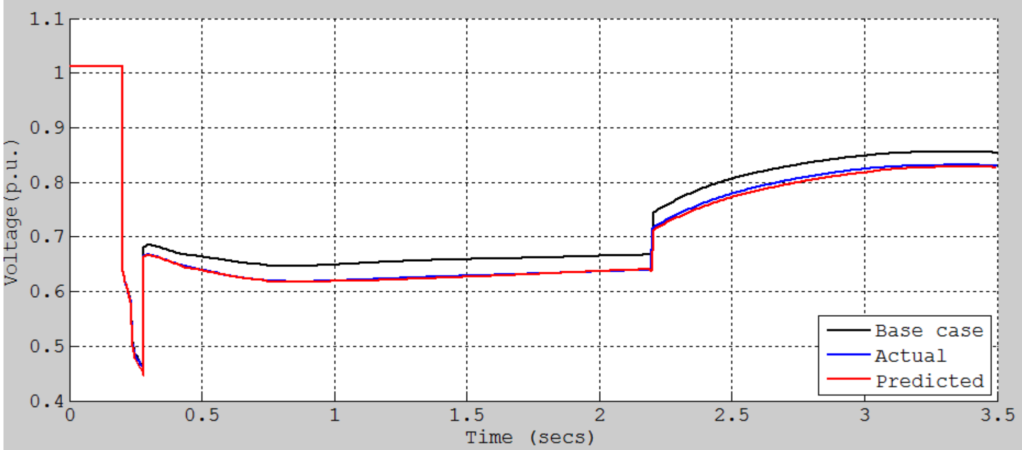

Figure 2: Actual and predicted voltage at bus 1 for a ΔKp of 5 at 20 buses

Figure 3 shows the actual and predicted bus voltages at bus 2 for a ΔK_p of 5 percent at the 20 study buses. Bus 2 is major system load bus, which is electrically far from the fault location. It can be seen that actual and predicted trajectories follow each other closely.

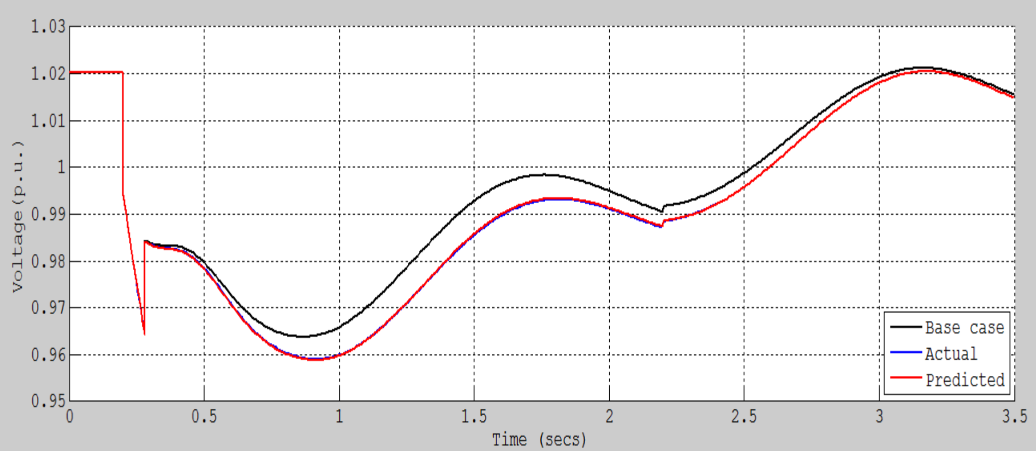

Figure 3: Actual and predicted voltage at bus 2 for a ΔKp of 5 at 20 buses

Relays, contactors and discontinuities in the load characteristics introduce severe non-linearity in the load models. Nonlinear models lead to approximation errors in a trajectory sensitivity based approach since the method relies on the assumption of linearity. The trajectory sensitivity based approach can be erroneous when the base case and the actual case to be predicted do not encounter and traverse the same switching surfaces [12]. As an example, the base case and actual case (to be predicted) should have same number of motors tripping and same number of ACs (stalling / restarting) to get an accurate linear prediction of trajectories. For the present scenario simulated, ACs account for majority of the load. Stalling of additional AC units has a pronounced effect on the linear approximation. Large, small and quick trip motors form a smaller portion of the load in the area under study. Due to their smaller percentage unequal tripping of motors, do not have a significant impact on the trajectories system wide. However, at buses where motors do not trip in the base case but do in the actual case, the error introduced is significant.

In this study, during the base case simulation, small motors where observed to trip due to under voltage at 36 buses. However, for a ΔK_p of 5 at 20 buses, the repeat simulation revealed 43 buses where small motors tripped due to under voltage. Figure 4 shows the actual and predicted bus voltages at bus 3 for a ΔK_p of 5 at the 20 buses under study. The small motors constitute 14 percent of the load at this bus. From Figure 4, it can be seen that unequal number of motors tripping in the base case and actual case introduces prediction error towards the end of the simulation. However, this effect is not pronounced at buses, which are electrically far from bus this load bus. Thus the error is somewhat localized.

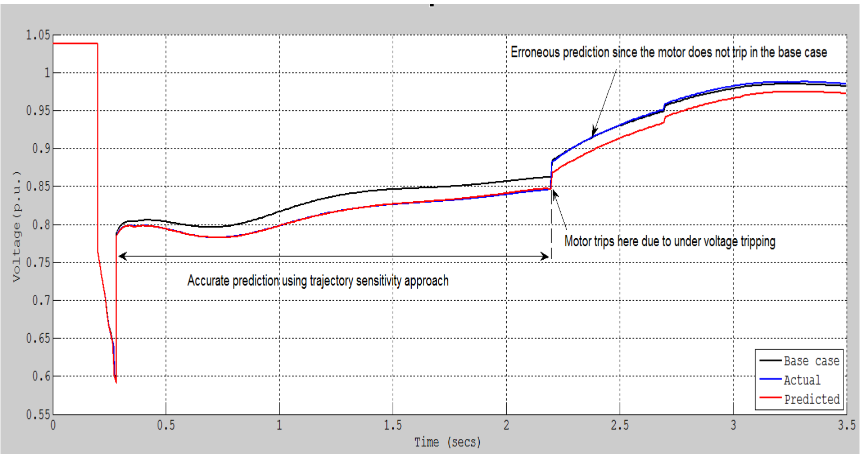

Figure 4: Actual and predicted voltage at bus 3 for 5 percent load change at 20 buses

For a better understanding of the effect mentioned above, a ΔK_p of 6 is considered at 20 buses. Stalling of AC units were observed on 20 buses in the base case. After a repeat simulation with a ΔK_p of 6 at 20 buses, stalling of AC units were observed on 27 buses. Bus 4 is a load bus where ACs stall in the actual case but not in the base case. Figure 5 shows the plot of actual and predicted voltage at bus 4 for a ΔK_p of 6 at 20 buses. Figure 5 shows that stalling of unequal number of ACs in the actual and base case introduces substantial error in the trajectory prediction. Figure 6 shows the actual and predicted voltage at bus 1, which is electrically far from bus 4. From Figure 6, it can be seen that unequal AC stalling introduces some but relatively minimal approximation error at this remote bus. Some further research is needed to look into possible ways to address such issues.

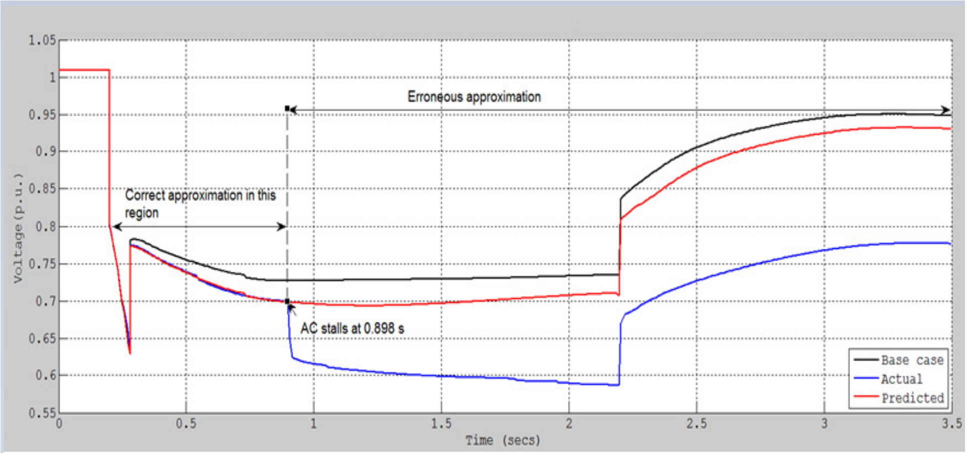

Figure 5: Actual and predicted voltage at bus 4 for 6 percent load change at 20 buses

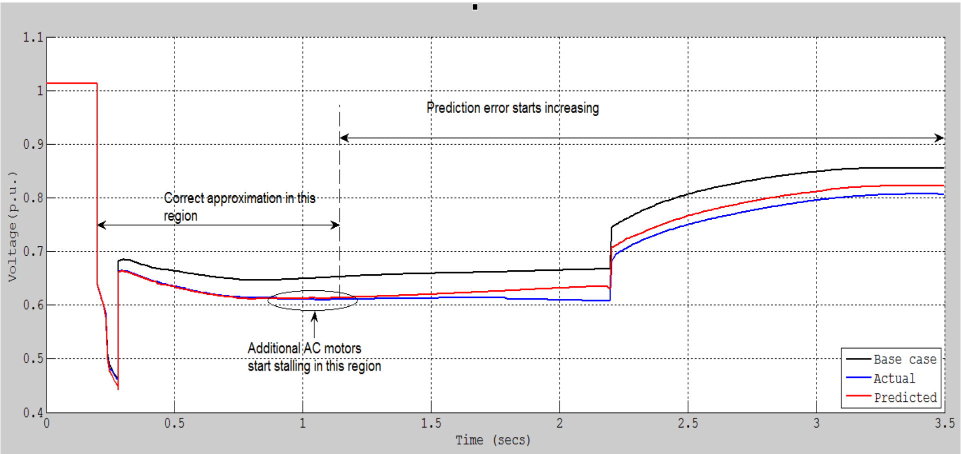

Figure 6: Actual and predicted voltage at bus 1 for 6 percent load change at 20 buses

Solution metrics for trajectory sensitivity analysis

The time requirements for performing the trajectory sensitivity analysis for the cases studied here are shown in column 2 of Table 2. The size of the file containing the Jacobian entries is 9 GB in this process. It should be noted that a trapezoidal method of integration is used in the trajectory sensitivity analysis. The increased simulation time is due to increased Newton-Raphson iterations required for convergence of solution during disturbances. To address the issue of increased time, an optimal multiplier can be used in conjunction with the Newton-Raphson update [15].

An optimal multiplier typically finds usage when the values of the variables oscillate near the final solution [15]. The problem is formulated as follows. The intermediate step of a time domain simulation involving differential-algebraic equations can be stated as a set of nonlinear equations given by

$$f(x) = 0 \tag{1.6}$$

The Newton's method transforms the problem in to a sequence of linear equations, whose solution approaches the solution of (1.6) iteratively. If the initial guess of the solution vector is x_0 then the linear problem is given by

$$-f(x_0) = J\Delta x \tag{1.7}$$

Where J is the Jacobian matrix containing the partial derivatives of f with respect to x and Δx is the correction vector by which x_0 is incremented. The next approximation of the solution x_1 is given by

$$x_1 = x_0 + \Delta x \tag{1.8}$$

This approach can be modified by introducing an optimal multiplier α, such the new estimate for the solution is given by

$$x_1 = x_0 + \alpha \Delta x \tag{1.9}$$

The value of α is calculated by solving a one dimensional minimization problem

$$\alpha = \arg\min(f(x_0 + \alpha\Delta x)^t f(x_0 + \alpha\Delta x)) \tag{1.10}$$

The exact solution of the one dimensional optimization problem in (1.10) is time consuming. For this study, a cubic interpolation technique has been used to find an inexact value of α. A detailed description of this approach can be found in [15]. The optimal multiplier is used only if the N-R routine does not converge in 3 iterations. The following observations were made after the introduction of the optimal multiplier α in the N– R routine (i) a reduction of more than 30 minutes in the total simulation time, (ii) a reduction in the stored data (Jacobian entries) from 9 GB to 5 GB, and (iii) a slight increase in error in the trajectory approximation. The trajectory sensitivity analysis of Case 1 and Case 2 were repeated using the optimal mal multiplier method. As an example, Figure 7 shows the actual and predicted bus voltages at the same bus as shown in Figure 2. As can be seen this approach introduces a small increase in the error in the sensitivity calculation. Table 2 shows the comparison of the computation time by both methods, it can be seen that a saving of about 30 minutes in computation time is achieved by introducing an optimal multiplier with a minimal additional error.

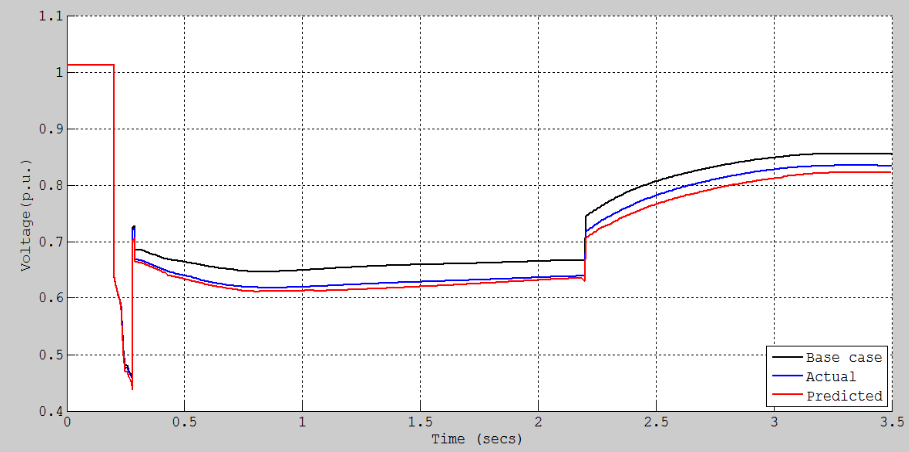

Figure 7: Actual and predicted voltage at bus 1 for 5 percent load change at 20 buses (N-R method with optimal multiplier)

Table 2: Comparison of simulation metrics

Routines	**Time (N-R without optimal**	**Time (N-R with optimal**
Time domain simulation (includes storing Jacobian)	3487.729 sec	1695.548 sec
Calculate initial values of sensitivities	1.708 sec	No change
Calculate the sensitivities	913.87 sec	353.97 sec
Create the final trajectory	32.29 sec	No change
Total time	4435.59 sec (74 mins)	2083.108 sec (35 mins)
Size of file containing Jacobian entries	9 GB	5 GB

5. CONCLUSIONS

Dynamic load modeling is becoming a more important aspect of performing time-domain stability studies for transmission planning. One of the key challenges in load modeling is determining the composition and aggregate model parameters. An approach to addressing the issue of uncertainties in the composition and/or model parameters is the application of load model sensitivity analysis. In this paper, an approach has been presented using trajectory sensitivity analysis. The main benefit of this method is that it allows a planner to study multiple scenarios with uncertain load parameters without the need of multiple simulations. Since, multiple sensitivities can be computed in parallel it enables additional savings in computational effort. The main disadvantage at present is that being a linear approach it cannot sufficiently handle severe non-linearity in load models. If the sensitivity cases push the system into additional motors stalling and/or tripping, the predicted trajectory can have significant errors associated with it. One other concern is the required computation time due to the implicit integration methods and the large overhead associated with calculating and storing the sensitivity data. However, it has been shown that at the expense of a relatively small additional error the computational time can be significantly reduced with an optimal multiplier in the integration algorithm. Further research will be needed to address the issues with non-linearities.

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