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Power System Dynamics Prediction with Measurement-Based AutoRegressive Model

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SUMMARY

The rapidly growing deployment of Phasor Measurement Units (PMU) makes it feasible to analyze power system dynamic behavior in a real-time environment, and even prediction or early detection of instability, using synchrophasor data. Autoregression is an important technique to extract relationships among measurement signals and is a powerful tool of system identification.

In this paper, a measurement-based Multivariate AutoRegressive (MAR) model is presented. It is designed to build a mathematical model for several measurement signals. The MAR model is a linear model. It is suitable for a linear condition which holds the majority of the time with power system operations. With a given model order and study signals, the MAR model can be trained to minimize overall error. Once it is trained, it can be used to predict the dynamics of study signals recursively with some initial dynamics data points.

The main challenge in training the MAR model is that the number of unknowns is proportional to the product of the signal number and the model order. Though more signals and higher order usually give a better prediction result, too many unknowns cannot be solved with a limited number of measurement data points. This problem is addressed in this paper with the delayed correlation coefficient technique. By analyzing the structure of the MAR model, the contribution of all input signals can be measured with a delayed correlation coefficient. Those signals with high correlation coefficients contribute more than signals with lower coefficients. To reduce the number of input terms, only the signals with high coefficients are chosen as inputs.

To verify the accuracy of the MAR model and model reduction technique, a 23-bus model is examined. Simulations show that the MAR model can provide an accurate prediction for events of a similar type. And model prediction can be dramatically reduced with the correlation coefficient technique while keeping model accuracy.

To conclude, the MAR model proposed in this paper is an attractive approach for power system dynamics prediction. It can be trained with pure measurement data. Though the MAR model is complicated for too many study signals, its complexity can be reduced with the delayed correlation coefficient technique without losing accuracy.

KEYWORDS

Power systems, Synchrophasor measurement, Multivariate AutoRegressive (MAR) model, Model reduction, Correlation coefficient

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I. INTRODUCTION

A power system is subjected to small or large disturbances at all times. According to power system reliability standards [1], major disturbances should be limited in a small area and cleared with relay devices. However, a power system can lose synchronism in some extreme cases such as cascading failures which may lead to disastrous social and economic consequences [2]. Predicting system dynamics after an event happens can help to assess and maintain the security of the grid. Physical model-based time-domain simulation is the primary method to assess system dynamic response under events[3]. Though the simulation-based method hass more detailed representation of the grid (compared to a measurement-based technique), and the ability to simulate "what-if" scenarios, including contingencies, it is difficult to update the model to the continuously evolving operating conditions, and simulation models, especially load models, are not accurate for on-line applications[4].

With the growing deployment of Phasor Measurement Units (PMU) [5] and Frequency Disturbance Recorders (FDRs) [6], a massive amount of measurement data of power system dynamics are recorded. The measured phasor data are more representative of actual system conditions than time-domain simulation results because they are obtained from directly measured voltage and current waveforms. The advantage of synchrophsor data reveals a possible application of studying power system with pure measurement data.

There are mainly two approaches to predict power system dynamics with measurement data. One is combining measurement data with partial model parameters, e.g., generator impedance, inertia, etc[7]-[8], the other is predicting dynamics with pure measurement data by treating a power system as a black-box with finite measurement points of PMU or FDR[9]-[11]. Artificial intelligence based methods also fall in the latter category[12].

This paper deals the dynamics prediction problem with system identification, and is organized as follows. In section II, a Multivariate AutoRegressive (MAR) model is introduced and the dynamics prediction procedure with the MAR model is proposed. Model training and reduction issues are discussed in section III. In section IV, some prediction examples are given and the model reduction technique is verified. Conclusions are drawn in section V.

II. THE PREDICTION MODEL WITH THE MULTIVARIATE AUTOREGRESSIVE MODEL

A. Multivariate AutoRegressive Model

System identification (SI) is an important method of building mathematical models for a dynamic system[13]. For a linear time-invariant system sampled at a time interval of h, a general SI transfer function structure is

$$A(q)y(t)=\frac{B(q)}{F(q)}u(t)+\frac{C(q)}{D(q)}e(t) \tag{1}$$

where

$$A(q)=1+a_1q^{-1}+...+a_{n_a}q^{-n_a}, B(q)=b_1q^{-1}+...+b_{n_b}q^{-n_b}, C(q)=1+c_1q^{-1}+...+c_{n_c}q^{-n_c}$$
$$D(q)=1+d_1q^{-1}+...+d_{n_d}q^{-n_d}, F(q)=1+f_1q^{-1}+...+f_{n_f}q^{-n_f}.$$

Here, $y(t)$ is the output signal which is the observable (measurable) system response of interest, $u(t)$ is the input signal which is a disturbance signal of the system or a stimuli manipulated by the observer, and $e(t)$ is a sequence of independent random variables. n_a, n_b, n_c, n_d, and n_f are orders of each part. q^{-1} is a backward shift operator and $q^{-1}y(t)= y(t-h)$.

For power systems, dynamics response of frequency, voltage magnitude and phase angle is of key interest and thus can be chosen as output signals of an observation model (1). Though they can be directly measured with PMUs or FDRs, the disturbance signals of the event, i.e., input signals, are hard to measure due to the diversity of events. Since power system dynamic quantities affect and are affected by other measurement signals, an observation model can be developed to model the interaction between those measurement signals by choosing one signal as an output signal and treating other signals as input signals.

With p interactional measurement signals $y_1(t)$, …, $y_p(t)$, if $y_i(t)$ is chosen as an output signal and all other signals are treated as input signals, a Multi-Input Single-Output (MISO) MAR model with $C(q)$=1, $D(q)$=1 and $F(q)$=1 can be developed as

$$A_i(q)\,y_i(t) = \sum_{j=1,\ j\neq i}^{p} B_{ij}(q)\,y_j(t) + e_i(t). \tag{2}$$

Expanding (2) yields

$$y_i(t) + \sum_{k=1}^{n_{ai}} a_{ik}\,y_i(t-kh) = \sum_{j=1,\ j\neq i}^{p} \sum_{k=1}^{n_{bji}} b_{ijk}\,y_j(t-kh) + e_i(t), \tag{3}$$

where n_{ai} is the order of signal $y_i(t)$ and n_{bji} is the order of signal $y_j(t)$ when $y_i(t)$ is the output signal and $y_j(t)$ is the input signal. k is the index of time delay.

In the MAR model, the orders of each signal can be different. For simplicity, a uniform order n can be chosen for all signals. With this manipulation, (3) can be rewritten as

$$y_i(t) = -\sum_{k=1}^{n} a_{ik}\,y_i(t-kh) + \sum_{j=1,\ j\neq i}^{p} \sum_{k=1}^{n} b_{ijk}\,y_j(t-kh) + e_i(t). \tag{4}$$

Let $b_{iik} = -a_{ik}$, we have

$$y_i(t) = \sum_{j=1}^{p} \sum_{k=1}^{n} b_{ijk}\,y_j(t-kh) + e_i(t). \tag{5}$$

Equation (5) can be further expressed in vector form as

$$y_i(t) = \sum_{j=1}^{p} \mathbf{b}_{ij}\mathbf{y}_j(t) + e_i(t) \tag{6}$$

or

$$y_i(t) = \sum_{j=1}^{p} \mathbf{y}_j(t)^T\,\mathbf{b}_{ij}^{\ T} + e_i(t), \tag{7}$$

where superscript ^{T} indicates transpose, and

$$\mathbf{b}_{ij} = \left[b_{ij1},\ldots,b_{ijn}\right], \mathbf{y}_j(t) = \left[y_j(t-h),\ldots,y_j(t-nh)\right]^T.$$

Choosing each measurement signal as output signal, we have

$$\begin{cases} y_1(t) = \sum_{j=1}^{p} \mathbf{b}_{1j}\mathbf{y}_j(t) + e_1(t) \\ \quad\cdots \\ y_p(t) = \sum_{j=1}^{p} \mathbf{b}_{pj}\mathbf{y}_j(t) + e_p(t) \end{cases}. \tag{8}$$

Let $\mathbf{y}(t) = \left[y_1(t),\ldots,y_p(t)\right]^T$, we have a Multi-Input Multi-Output (MIMO) MAR model

$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{b}_{11} & \cdots & \mathbf{b}_{1p} \\ \cdots & \cdots & \cdots \\ \mathbf{b}_{p1} & \cdots & \mathbf{b}_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{y}_1(t) \\ \cdots \\ \mathbf{y}_p(t) \end{bmatrix} + \begin{bmatrix} e_1(t) \\ \cdots \\ e_p(t) \end{bmatrix}. \tag{9}$$

Since the MISO MAR model (7) is part of the MIMO MAR model (9), (7) can be treated as a sub-model of MIMO MAR model (9) and is denoted as "sub-model i".

B. Prediction Procedure

In (3), the time delay index k starts from 1 and $y_i(t)$ is totally determined by historical data, i.e., data at time prior to time t. So, equation (9) give a one-step prediction of a dynamic system. In equation (9), the random part $e(t)$ corresponds to the unmeasured disturbance signals and is difficult to model. For simplicity, the $e_i(t)$ part is neglected and the MAR model becomes

$$\mathbf{y}(t)=\begin{bmatrix}\mathbf{b}_{11} & \dots & \mathbf{b}_{1p}\\ \dots & \dots & \dots \\ \mathbf{b}_{p1} & \dots & \mathbf{b}_{pp}\end{bmatrix}\begin{bmatrix}\mathbf{y}_1(t)\\ \dots \\ \mathbf{y}_p(t)\end{bmatrix} \tag{10}$$

or

$$\mathbf{y}(t)=\mathbf{BY}(t) \tag{11}$$

where

$$\mathbf{B}=\begin{bmatrix}\mathbf{b}_{11} & \dots & \mathbf{b}_{1p}\\ \dots & \dots & \dots \\ \mathbf{b}_{p1} & \dots & \mathbf{b}_{pp}\end{bmatrix}, \qquad \mathbf{Y}(t)=\left[\mathbf{y}_1(t)^T \quad \dots \quad \mathbf{y}_p(t)^T\right]^T.$$

Once the MAR model is obtained, dynamics can be predicted recursively based on the flowchart in Fig. 1 where t_{max} is the length of a study time window.

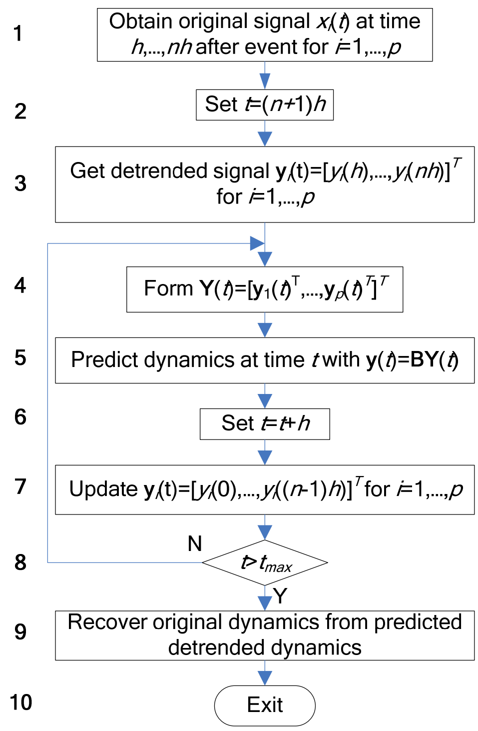


Fig.1 Flowchart of dynamics prediction with MAR model

III. Model Training and Reduction

C. Training of the MAR Model

The MIMO MAR model (9) is comprised of p sub-models, and can be developed by training each sub-model separately. For sub-model i of (7), m-n equations can be written as (12) with an event of m measurement data points.

$$\begin{cases} y_i\left((n+1)h\right)=\sum_{j=1}^{p}\mathbf{y}_j\left((n+1)h\right)^T\mathbf{b}_{ij}{}^T+e_i\left((n+1)h\right) \\ \qquad \dots \\ y_i(mh)=\sum_{j=1}^{p}\mathbf{y}_j(mh)^T\mathbf{b}_{ij}{}^T+e_i(mh)\end{cases} \tag{12}$$

It can be re-written as

$$\mathbf{Y}_i = \mathbf{A}_i\mathbf{B}_i + \mathbf{E}_i, \tag{13}$$

where

$$\mathbf{A}_i = \begin{bmatrix} \mathbf{y}_1((n+1)h)^T & \dots & \mathbf{y}_p((n+1)h)^T \\ \dots & \dots & \dots \\ \mathbf{y}_1(mh)^T & \dots & \mathbf{y}_p(mh)^T \end{bmatrix}, \quad \mathbf{Y}_i = \left[y_i((n+1)h) \ \dots \ y_i(mh) \right]^T.$$

$$\mathbf{B}_i = \left[\mathbf{b}_{i1} \ \dots \ \mathbf{b}_{ip} \right]^T, \qquad \mathbf{E}_i = \left[e_i((n+1)h) \ \dots \ e_i(mh) \right]^T$$

To get the best MAR model with (13), the error part $\mathbf{E}_i$ should be minimized. Parameter $\mathbf{B}_i$ can be estimated with least square error estimator to minimize the error part $\mathbf{E}_i$,

$$\mathbf{B}_i = \left(\mathbf{A}_i^T\mathbf{A}_i\right)^{-1}\mathbf{A}_i^T\mathbf{Y}_i. \tag{14}$$

D. Reduction of MAR Model

The MISO MAR model developed with (7) includes *n* parameters for each input signal y_j. It is called full MAR model with *np* unknowns. With more and more measurement units deployed, the number of unknowns increases linearly with measurement number *p*. Though the MAR model is linear, too many unknowns may cause huge computation burden and model complexity, and thus prevent the full MAR model from being applicable for online applications.

The increase of number of unknowns leads to a possible problem of under-determinacy. There are *m-n* equations (constraints) but *np* unknowns in (7) for a MISO MAR model. If $m-n<np$ (or $m-n\geq np$ and $\mathbf{A}_i$ is singular), (12) is underdetermined and mathematically unsolvable. To overcome this difficulty, either the data point number should be increased or the number of unknowns should be reduced.

However, *m* cannot be increased arbitrarily. In most cases, a disturbance is damped in 20s or shorter. The time window length for model training is usually 10s to 20s. Too long of a time window may include data points of the new steady state and make $\mathbf{A}_i$ singular. Under a specific PMU reporting rate, the data length *m* is limited. So, *m* cannot be increased unless PMU is reporting at a higher rate. It is more practical to reduce the number of unknowns than to increase the measurement data length.

Since the number of unknowns is *np*, there are two direct ways to reduce the complexity of a MISO MAR model. One way is to reduce the model order *n*. However, *n* is a key parameter of the MAR model, and dynamics prediction accuracy is greatly affected by the model order. In general, a higher order usually yields better training accuracy and prediction accuracy. To keep high prediction accuracy, the model order cannot be dramatically reduced.

The other way is to reduce the number of input signals. The relationship between the output signal y_i and different input signals y_j is different. Those input signals which are more related to the output signal contribute more to the output signal. To measure the contribution of each input signal, a zero-delay Correlation Coefficient (CC) is introduced in [14] to select the input signals. The zero-delay CC is defined as

$$r_{ij} = \left| \frac{\sum_{l=1}^{m} y_i(lh)\, y_j(lh)}{\sqrt{\sum_{l=1}^{m} y_i(lh)^2}\sqrt{\sum_{l=1}^{m} y_j(lh)^2}} \right|. \tag{15}$$

With the method proposed in [14], input signals with high zero-delay CC are selected as effective input signals and other signals are completely removed from (7).

The main problem of this method is that it does not differentiate the input signals and specific input terms of each input signal. For each input signal y_j, there are *n* input terms of output signal y_i in (7), i.e., $b_{ij1}y_j(t\text{-}h)$, $b_{ij2}y_j(t\text{-}2h)$,…,and $b_{ijn}y_j(t\text{-}nh)$. Those input terms are the delayed time series of original signal y_j. If the zero-delay CC r_{ij} is high, it is not necessary that all the n delayed input terms of y_j contribute significantly to output signal y_i, and vice versa. It is preferable to study the contribution of each input term specifically, instead of the overall contribution of each input signal.

To overcome the drawbacks of the zero-delay CC, the delayed CC of each input term is used in this paper to find a better combination of input terms. Take a k-delay input term of signal y_j, i.e., $y_j(t\text{-}kh)$ of (7), for example, the k-delayed CC between the input signal y_j and output signal y_i is

$$r_{ijk}=\left|\frac{\sum_{l=k+1}^{m} y_i(lh)\, y_j((l-k)h)}{\sqrt{\sum_{l=k+1}^{m} y_i(lh)^2}\sqrt{\sum_{l=1}^{m} y_j((l-k)h)^2}}\right|. \tag{16}$$

The delayed CC examines the correlation of each delayed input term with the output signal. Comparing (15) with (16), it can be found that the zero-delay CC is symmetric, i.e., $r_{ij}=r_{ji}$, however, the delayed CC is asymmetric, i.e., $r_{ijk}\neq r_{jik}$. This leads to the main difference between the input selections with the two CCs. With zero-delay CC, if signal y_j is highly correlated with signal y_i, signal y_j is the input signal of signal y_i, and vice versa. For delayed CC, if k-delay input term of signal y_j is highly correlated with signal y_i, $y_j(t\text{-}kh)$ can be selected as the input term of signal y_i. However, if the k-delay input term of signal y_i is not highly correlated with signal y_j, it does not have to be selected as an input term of y_j. So, **B** of (11) does not necessarily have symmetric structure when delayed CC is used for input term selection.

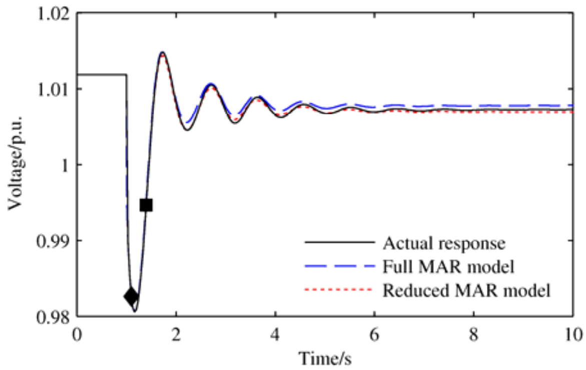
With the delayed CC, it is not necessary to find the optimal model order n. With a higher model n, all input terms can be examined with the delayed CC and only those with the delayed CC higher than a user-defined threshold ε are selected for developing the sub-model of y_i. The delayed CC lies in the range of [0, 1] and a threshold higher than 0.9 is usually suggested for input term selection. This will be further examined in section IV.

IV. Examples

A 23-bus model is chosen as a test system. The system data can be found in the PSS/E manual[15]. The simulation time step is set as half cycle (1/120s). Frequency, voltage and phase angle of all 17 PQ buses are monitored. In this section, a full MAR model of those 51 signals is developed with a 15s-long data, under 1% load increase event and prediction is made for a 5% load increase event. The order of each signal is 13. Additionally, suppose the data reporting rate of 23-bus model is 30Hz, which can be achieved by down sampling the 120Hz simulation data, another reduced MAR model is also developed with 15s-long dynamic data with threshold of delayed CC of 0.9. Prediction of voltage and frequency of bus 151 are shown in Fig. 2 and 3.

In Fig. 2 and 3, diamonds on the dashed lines show where prediction of the full MAR model starts when data rate is 120Hz. Comparing the dashed lines with actual response, it can be concluded that the full MAR model can predict the dynamics with high accuracy.

The squares in Fig. 2 and 3 show where prediction of the reduced MAR model starts when data rate is 30Hz. In the reduced MAR model, there are only 44 unknowns for the submodel of signal 1 (frequency of bus 151). The submodel with the most unknowns is signal 15 (phase angle of bus 201), which is constituted by 190 unknowns. The submodel with the most unknowns is signal 14 (voltage of bus 201), which consists of 35 unknowns. The number of unknowns of the reduced MIMO MAR model is 5030 which is about 15% of the full MAR model. Though the MAR model is dramatically reduced, it still provides good prediction result in Fig. 2 and 3.



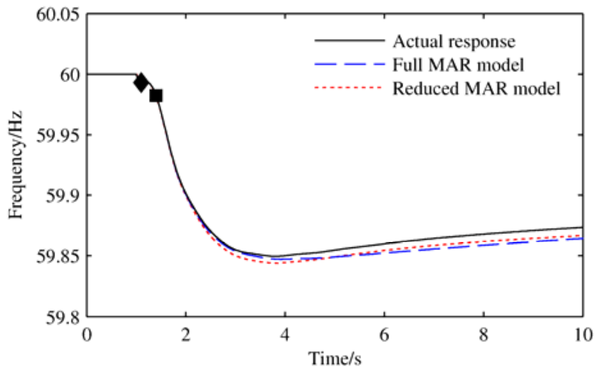


Fig.2 Frequency dynamics prediction with reduced model Fig.3 Frequency prediction with reduced model

V. CONCLUSIONS

With more and more PMUs, massive amounts of measurement data give an alternative approach to study power system dynamics with real response. The MAR model proposed in this paper is promising for power system study of linear conditions with pure measurement data. It can be easily updated online with typical events. To improve its feasibility, a model reduction technique with delayed correlation coefficient provides a good guideline for input selections. Simulations show that the accuracy of prediction is high for test cases and the model reduction technique works well to select the most effective input terms.

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